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Title Key: Inventory Model of Deteriorating Products With…

Inventory Model of Deteriorating Products with Non-Uniform Demand Rates Under Life Time and Trade Credit.

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Abstract:
In this paper, an inventory model has been developed for deteriorating products with life time, inventory level dependent demand rate and time dependent demand rate under trade credit. Three components demand rate has been considered. Deterioration rate has been taken constant. Permissible delay in payments is allowed.

1. Introduction:

In traditional inventory models the demand rate is assumed to be constant. But it is observed that the stock level may influence the demand rate in the case of some consumer products. It is common experience that displaced stock level attracts more consumers. This observation inspired the researchers to consider the stock dependent demand rate.

Gupta & Vrat (1986) discussed the stock dependent demand rate inventory model. Calculation of average system cost was not correct in this paper. **Mandal & Phaujdar** (1989) suggested the correction to the average system cost. **Bakar and Urban** (1988) provided the first rigorous attempt in developing the stock dependent demand rate. The functional form presented by them is realistic and logical from practical as well as economic point of view.

Dutta & Pal (1990) developed an inventory model with stock dependent demand rate using some functional form as taken by **Bakar and Urban** (1988). **Dutta & Pal** (1990) developed another model for deteriorating items with demand rate dependent on inventory level with shortages. **Mandal & Phaujdar** (1989) also developed another model for deteriorating items with stock dependent demand rate, variable rate of deterioration and shortages fully backlogged. **Sarkar, et al.** (1997) introduced the realistic concept of decreasing demand.

Jamal, et al. (1997) developed an EOQ model for deteriorating items allowing shortages. **Subbiah et al.** (2004) developed an inventory model with stock dependent demand. **Teng et al.** (2005) discussed an EPQ model for deteriorating products with price and stock dependent demand. **Basu & Sinha** (2007) presented an ordering policy for deteriorating products with two component demand and price break allowing shortages. **Chakarvarti and Sen** (2008) presented an inventory model with variable rate of deteriorationand alternating replenishing rates taking shortages into consideration**.** **Kharna et al.** (2010) developed an EOQ model with price and stock demand rate. **Kundu et al.** (2013) developed an EOQ model for time dependent deteriorating items with alternating demand rates allowing shortages and and time discount. **Hari Kishan, Megha Rani and Deep Shikha** (2012) discussed the inventory model of deteriorating products with life time under declining demand and permissible delay in payment.

Notations and Assumptions:

Notations:

The following notations have been used in this paper:

$q(t)$: Inventory level at any time t.

S: Stock level at the beginning of each cycle after fulfilling the backorders.

Q: Stock level at the beginning of each cycle plus the amount of shortage.

A: Ordering cost per order.

μ: Time at which deterioration of items starts.

T_0: Time at which shortage starts.

T: Length of cycle time.

C_H: The total holding cost of inventory in the interval $[0, T_0]$.

C_S: The total shortage cost of inventory in the interval $[T_0, T]$.

C_D: The total holding cost of inventory in the interval $[\mu, T_0]$.

Assumptions:

1. There is single item in the inventory system.
2. Lead time is negligible. It is considered as zero.
3. Replenishments are instantaneous, i.e. the replenishment rate is taken infinite.
4. Shortages are allowed and fully backlogged.
5. Three components demand rate is deterministic and known function of instantaneous inventory level during the life time, constant during the deteriorating period and time dependent during the shortage period. Thus the demand rate is given by

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$$D = \begin{cases} \alpha q^{\beta}, 0 \le t \le \mu \\ \alpha, \quad \mu \le t \le T_0 \\ (\alpha - \gamma(t - T_0)), T_0 \le t \le T \end{cases}$$

Mathematical Model and Analysis:

Let Q be the number of items received at the beginning of the cycle. $(Q - S)$ items are delivered for the fulfillment of backorder leaving a balance of S as the initial inventory of new cycle. The inventory level depleted at a rate of αq^{β} during the period $[0, \mu]$. During the period $[\mu, T_0]$ the inventory level depleted at the rate α. The inventory level falls to zero at time T_0. Shortages are allowed for replenishment upto time T. The inventory system is governed by the following differential equations:

$$\frac{dq}{dt} = -\alpha q^{\beta}, \qquad 0 \le t \le \mu \qquad \ldots(1)$$

$$\frac{dq}{dt} + \theta t q = -\alpha, \qquad \mu \le t \le T_0 \qquad \ldots(2)$$

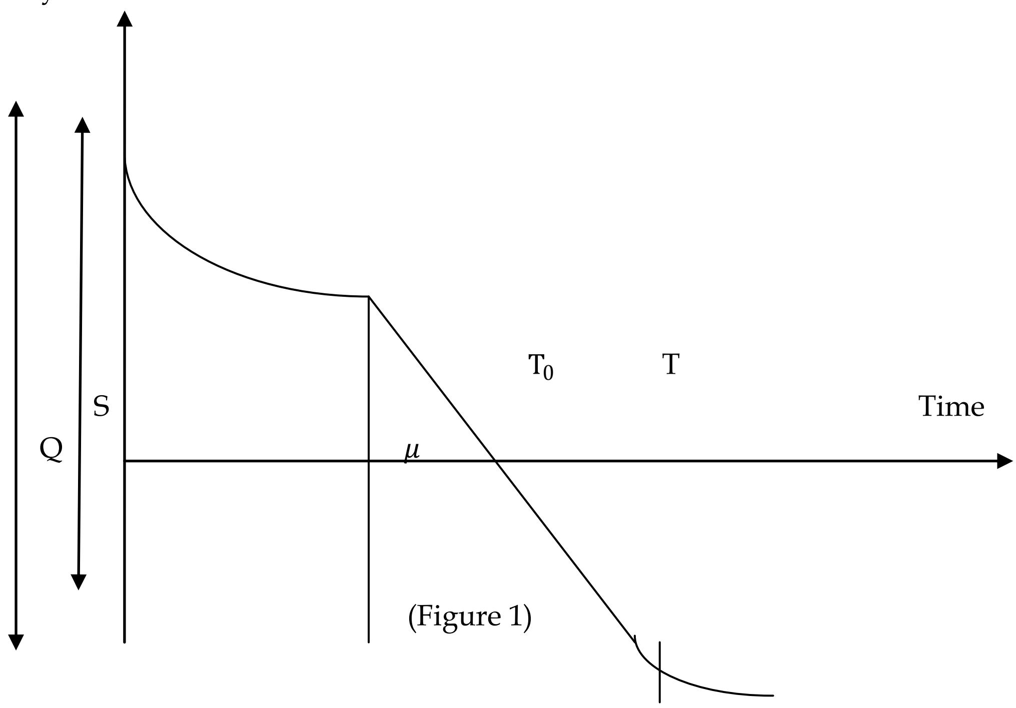
$$\frac{dq}{dt} = -(\alpha - \gamma(t - T_0)), \qquad T_0 \le t \le T \qquad \ldots(3)$$

With the following boundary conditions:

$$q(0) = S, q(T_0) = 0 \text{ and } q(T) = -(Q - S). \qquad \ldots(4)$$

This model is shown in the figure given below:

Inventory



(Figure 1)

Solution of equations (1), (2) and (3) with the help of boundary conditions (4) are respectively given by

$$q = S[1-\alpha p t]^{\frac{1}{p}}, \qquad 0 \le t \le \mu \qquad \ldots(5)$$

$$q = \alpha T_0 + \frac{\alpha\theta T_0^3}{6} - \alpha t + \frac{\alpha\theta t^3}{3} - \frac{\alpha\theta T_0 t^2}{3}, \qquad \mu \le t \le T_0 \qquad \ldots(6)$$

$$q = \alpha(T_0 - t) + \gamma T_0 t + \frac{\gamma}{2}(T_0^2 + t^2), \qquad T_0 \le t \le T \qquad \ldots(7)$$

where $p = 1-\beta$.

From expressions (5) and (6), we get

$$S = \frac{\alpha T_0 + \frac{\alpha\theta T_0^3}{6} - \alpha t + \frac{\alpha\theta\mu^3}{3} - \frac{\alpha\theta T_0\mu^2}{3}}{[1-\alpha p\mu]^{\frac{1}{p}}}. \qquad \ldots(8)$$

Annual ordering cost=$\frac{A}{T}$.

The total holding cost during the inventory cycle is given by

$$C_H = c_1\left[\int_0^{T_0} q\,dt\right]$$

$$= c_1\left[\int_0^{\mu} q\,dt + \int_{\mu}^{T_0} q\,dt\right]$$

$$= c_1\Big[\frac{S}{\alpha(p+1)}[1-\alpha p\mu]^{\frac{p+1}{p}} + \frac{\alpha T_0^2}{2} + \frac{5\alpha\theta T_0^4}{36} - \alpha T_0\mu - \alpha\theta T_0^3\mu$$

$$+ \frac{\alpha\mu^2}{2} + \frac{\alpha\theta T_0\mu^3}{9} - \frac{\alpha\theta\mu^4}{12}\Big]. \qquad \ldots(9)$$

Total shortage cost for the entire cycle is

$$C_S = c_2\left[\int_{T_0}^{T} q\,dt\right]$$

$$= c_2\left[\left[\alpha\left(T_0 T - \frac{T^2}{2}\right) + \gamma T_0\frac{T^2}{2} + \frac{\gamma}{2}\left(T_0^2 T + \frac{T^3}{3}\right)\right]\right.$$

$$\left. -\alpha\left(T_0\mu - \frac{\mu^2}{2}\right) + \gamma T_0\frac{\mu^2}{2} + \frac{\gamma}{2}\left(T_0^2\mu + \frac{\mu^3}{3}\right)\right]. \qquad \ldots(10)$$

Total deteriorated units per cycle are given by

$$D = S - \int_{\mu}^{T_0}(\alpha + \gamma q)dt$$

$$= S - \alpha T_0 - \frac{\alpha\gamma T_0^2}{2} - \frac{5\alpha\gamma\theta T_0^4}{36} + \left(\alpha + \alpha\gamma T_0 + \frac{\alpha\gamma\theta T_0^3}{6}\right)\mu$$

$$- \frac{\alpha\gamma\mu^2}{2} + \frac{\alpha\theta T_0\mu^3}{9} - \frac{\alpha\theta\gamma\mu^4}{12}. \qquad \ldots(11)$$

The total deterioration cost is given by

$$C_D = c\Big[S - \alpha T_0 - \frac{\alpha\gamma T_0^2}{2} - \frac{5\alpha\gamma\theta T_0^4}{36} + \left(\alpha + \alpha\gamma T_0 + \frac{\alpha\gamma\theta T_0^3}{6}\right)\mu - \frac{\alpha\gamma\mu^2}{2} + \frac{\alpha\theta T_0\mu^3}{9} - \frac{\alpha\theta\gamma\mu^4}{12}\Big]. \quad \ldots(12)$$

The total amount backordered at the end of each cycle is given by

$$Q - S = \alpha(T_0 - t) + \gamma T_0 t + \frac{\gamma}{2}(T_0^2 + t^2). \quad \ldots(13)$$

Now we consider the following two cases:

Case 1: $M \le T_0$.

Sub case I: $0 < M \le \mu$.

In this case, the interest payable is given by

$$I_{p1}^1 = cI_c\left[\int_M^\mu q(t)\,dt + \int_\mu^{T_0} q(t)\,dt\right]$$
$$= cI_c\left[\int_M^\mu S[1-\alpha pt]^{\frac{1}{p}}dt + \int_\mu^{T_0}\left(\alpha T_0 + \frac{\alpha\theta T_0^3}{6} - \alpha t + \frac{\alpha\theta t^3}{3} - \frac{\alpha\theta T_0 t^2}{3}\right)dt\right]$$
$$= cI_c\Big[\frac{S}{\alpha(p+1)}\left[(1-\alpha pM)^{\frac{p+1}{p}} - (1-\alpha p\mu)^{\frac{p+1}{p}}\right] + \frac{\alpha T_0^2}{2} + \frac{5\alpha\theta T_0^3}{36} - \left(\alpha T_0 + \frac{\alpha\theta T_0^3}{6}\right)\mu + \frac{\alpha\mu^2}{2} - \frac{\alpha\theta\mu^4}{12} + \frac{\alpha\theta T_0\mu^3}{9}\Big]. \quad \ldots(14)$$

In this case, the interest earned is given by

$$I_{e1}^1 = sI_e\left[\int_0^\mu \alpha q^\beta t dt + \int_\mu^{T_0} \alpha t dt\right]$$
$$= sI_e\left[-\frac{\mu(1-\alpha p\mu)^{\frac{\beta+p}{p}}}{(\beta+p)} + \frac{\left\{1-(1-\alpha p\mu)^{\frac{\beta+2p}{p}}\right\}}{\alpha(\beta+p)(\beta+2p)} + \frac{\alpha}{2}(T_0^2 - \mu^2)\right]. \quad \ldots(15)$$

Therefore the total cost per unit time is given by

$$TC(1) = \frac{1}{T}[A + C_H + C_S + C_D + I_{p1}^1 - I_{e1}^1]$$
$$= \frac{A}{T} + \frac{c_1}{T}\Big[\frac{S}{\alpha(p+1)}[1-\alpha p\mu]^{\frac{p+1}{p}} + \frac{\alpha T_0^2}{2} + \frac{5\alpha\theta T_0^4}{36} - \alpha T_0\mu - \alpha\theta T_0^3\mu + \frac{\alpha\mu^2}{2} + \frac{\alpha\theta T_0\mu^3}{9} - \frac{\alpha\theta\mu^4}{12}\Big] + \frac{c_2}{T}\left[\left[\alpha\left(T_0T - \frac{T^2}{2}\right) + \gamma T_0\frac{T^2}{2} + \frac{\gamma}{2}\left(T_0^2T + \frac{T^3}{3}\right)\right] - \alpha\left(T_0\mu - \frac{\mu^2}{2}\right) + \gamma T_0\frac{\mu^2}{2} + \frac{\gamma}{2}\left(T_0^2\mu + \frac{\mu^3}{3}\right)\right]$$
$$+ \frac{c}{T}\Big[S - \alpha T_0 - \frac{\alpha\gamma T_0^2}{2} - \frac{5\alpha\gamma\theta T_0^4}{36} + \left(\alpha + \alpha\gamma T_0 + \frac{\alpha\gamma\theta T_0^3}{6}\right)\mu$$

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$$-\frac{\alpha\gamma\mu^2}{2}+\frac{\alpha\theta T_0\mu^3}{9}-\frac{\alpha\theta\gamma\mu^4}{12}\Big]+\frac{cI_c}{T}\Big[\frac{S}{\alpha(p+1)}\Big[(1-\alpha pM)^{\frac{p+1}{p}}-(1-\alpha p\mu)^{\frac{p+1}{p}}\Big]$$
$$+\frac{\alpha T_0^2}{2}+\frac{5\alpha\theta T_0^3}{36}-\left(\alpha T_0+\frac{\alpha\theta T_0^3}{6}\right)\mu+\frac{\alpha\mu^2}{2}-\frac{\alpha\theta\mu^4}{12}+\frac{\alpha\theta T_0\mu^3}{9}\Big]$$
$$-\frac{sI_e}{T}\left[-\frac{\mu(1-\alpha p\mu)^{\frac{\beta+p}{p}}}{(\beta+p)}+\frac{\left\{1-(1-\alpha p\mu)^{\frac{\beta+2p}{p}}\right\}}{\alpha(\beta+p)(\beta+2p)}+\frac{\alpha}{2}(T_0^2-\mu^2)\right]. \quad \ldots(16)$$

Sub case II: $\mu < M \leq T_0$.
In this case, the interest payable is given by

$$I_{p1}^2 = cI_c\left[\int_M^{T_0} q(t)\right]$$

$$= cI_c\left[\int_M^{T_0}\left(\alpha T_0+\frac{\alpha\theta T_0^3}{6}-\alpha t+\frac{\alpha\theta t^3}{3}-\frac{\alpha\theta T_0t^2}{3}\right)dt\right]$$
$$= cI_c\left[\frac{\alpha T_0^2}{2}+\frac{5\alpha\theta T_0^3}{36}-\left(\alpha T_0+\frac{\alpha\theta T_0^3}{6}\right)\mu+\frac{\alpha\mu^2}{2}-\frac{\alpha\theta\mu^4}{12}+\frac{\alpha\theta T_0\mu^3}{9}\right]. \quad \ldots(17)$$

In this case, the interest earned is given by

$$I_{e1}^2 = sI_e\left[\int_0^{\mu}\alpha q^{\beta}tdt+\int_{\mu}^{T_0}\alpha tdt\right]$$
$$= sI_e\left[-\frac{\mu(1-\alpha p\mu)^{\frac{\beta+p}{p}}}{(\beta+p)}+\frac{\left\{1-(1-\alpha p\mu)^{\frac{\beta+2p}{p}}\right\}}{\alpha(\beta+p)(\beta+2p)}+\frac{\alpha}{2}(T_0^2-\mu^2)\right]. \quad \ldots(18)$$

Therefore the total cost per unit time is given by

$$TC(2) = \frac{1}{T}[A+C_H+C_S+C_D+I_{p1}^1-I_{e1}^1]$$
$$= \frac{A}{T}+\frac{c_1}{T}\Big[\frac{S}{\alpha(p+1)}[1-\alpha p\mu]^{\frac{p+1}{p}}+\frac{\alpha T_0^2}{2}+\frac{5\alpha\theta T_0^4}{36}-\alpha T_0\mu-\alpha\theta T_0^3\mu$$
$$+\frac{\alpha\mu^2}{2}+\frac{\alpha\theta T_0\mu^3}{9}-\frac{\alpha\theta\mu^4}{12}\Big]+\frac{c_2}{T}\left[\left[\alpha\left(T_0T-\frac{T^2}{2}\right)+\gamma T_0\frac{T^2}{2}+\frac{\gamma}{2}\left(T_0^2T+\frac{T^3}{3}\right)\right]\right.$$
$$\left.-\alpha\left(T_0\mu-\frac{\mu^2}{2}\right)+\gamma T_0\frac{\mu^2}{2}+\frac{\gamma}{2}\left(T_0^2\mu+\frac{\mu^3}{3}\right)\right]$$
$$+\frac{c}{T}\left[S-\alpha T_0-\frac{\alpha\gamma T_0^2}{2}-\frac{5\alpha\gamma\theta T_0^4}{36}+\left(\alpha+\alpha\gamma T_0+\frac{\alpha\gamma\theta T_0^3}{6}\right)\mu\right.$$
$$\left.-\frac{\alpha\gamma\mu^2}{2}+\frac{\alpha\theta T_0\mu^3}{9}-\frac{\alpha\theta\gamma\mu^4}{12}\right]$$
$$+\frac{cI_c}{T}\left[\frac{\alpha T_0^2}{2}+\frac{5\alpha\theta T_0^3}{36}-\left(\alpha T_0+\frac{\alpha\theta T_0^3}{6}\right)\mu+\frac{\alpha\mu^2}{2}-\frac{\alpha\theta\mu^4}{12}+\frac{\alpha\theta T_0\mu^3}{9}\right]$$

$$-\frac{sI_e}{T}\left[-\frac{\mu(1-\alpha p\mu)^{\frac{\beta+p}{p}}}{(\beta+p)}+\frac{\left\{1-(1-\alpha p\mu)^{\frac{\beta+2p}{p}}\right\}}{\alpha(\beta+p)(\beta+2p)}+\frac{\alpha}{2}(T_0^2-\mu^2)\right]. \quad \ldots(19)$$

Case 2: $M \geq T_0$.

In this case, there will be no interest charged.

The interest earned is given by

$$I_{e2}^1 = sI_e\left[\int_0^{\mu}\alpha q^{\beta}tdt+\int_{\mu}^{T_0}\alpha tdt+(M-T_0)\int_{T_0}^{M}(\alpha-\gamma(t-T_0))tdt\right]$$

$$= sI_e\left[-\frac{\mu(1-\alpha p\mu)^{\frac{\beta+p}{p}}}{(\beta+p)}+\frac{\left\{1-(1-\alpha p\mu)^{\frac{\beta+2p}{p}}\right\}}{\alpha(\beta+p)(\beta+2p)}+\frac{\alpha}{2}(T_0^2-\mu^2)\right.$$

$$\left.+(M-T_0)^2\left\{\frac{(\alpha+\gamma T_0)((M+T_0))}{2}-\frac{\gamma}{3}(M^2+MT_0+T_0^2)\right\}\right]. \quad \ldots(20)$$

The total cost per unit time is given by

$$TC(3)=\frac{1}{T}[A+C_H+C_S+C_D-I_{e2}^1]$$

$$=\frac{A}{T}+\frac{c_1}{T}\left[\frac{S}{\alpha(p+1)}[1-\alpha p\mu]^{\frac{p+1}{p}}+\frac{\alpha T_0^2}{2}+\frac{5\alpha\theta T_0^4}{36}-\alpha T_0\mu-\alpha\theta T_0^3\mu\right.$$

$$\left.+\frac{\alpha\mu^2}{2}+\frac{\alpha\theta T_0\mu^3}{9}-\frac{\alpha\theta\mu^4}{12}\right]+\frac{c_2}{T}\left[\left[\alpha\left(T_0T-\frac{T^2}{2}\right)+\gamma T_0\frac{T^2}{2}+\frac{\gamma}{2}\left(T_0^2T+\frac{T^3}{3}\right)\right]\right.$$

$$\left.-\alpha\left(T_0\mu-\frac{\mu^2}{2}\right)+\gamma T_0\frac{\mu^2}{2}+\frac{\gamma}{2}\left(T_0^2\mu+\frac{\mu^3}{3}\right)\right]$$

$$+\frac{c}{T}\left[S-\alpha T_0-\frac{\alpha\gamma T_0^2}{2}-\frac{5\alpha\gamma\theta T_0^4}{36}+\left(\alpha+\alpha\gamma T_0+\frac{\alpha\gamma\theta T_0^3}{6}\right)\mu\right.$$

$$\left.-\frac{\alpha\gamma\mu^2}{2}+\frac{\alpha\theta T_0\mu^3}{9}-\frac{\alpha\theta\gamma\mu^4}{12}\right]$$

$$-\frac{sI_e}{T}\left[-\frac{\mu(1-\alpha p\mu)^{\frac{\beta+p}{p}}}{(\beta+p)}+\frac{\left\{1-(1-\alpha p\mu)^{\frac{\beta+2p}{p}}\right\}}{\alpha(\beta+p)(\beta+2p)}+\frac{\alpha}{2}(T_0^2-\mu^2)\right.$$

$$\left.+(M-T_0)^2\left\{\frac{(\alpha+\gamma T_0)((M+T_0))}{2}-\frac{\gamma}{3}(M^2+MT_0+T_0^2)\right\}\right]. \quad \ldots(21)$$

Thus we have

$$TC=\begin{cases}TC(1), & 0<M\leq\mu\\ TC(2), & \mu<M\leq T_0\\ TC(3), & M\geq T_0\end{cases}$$

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For optimization, we have

$$\frac{\partial TC}{\partial T_0}=0, \frac{\partial TC}{\partial T}=0 \text{ and } \left(\frac{\partial^2 TC}{\partial {T_0}^2}\right).\left(\frac{\partial^2 TC}{\partial T^2}\right)-\left(\frac{\partial^2 TC}{\partial T_0 \partial T}\right)^2 \geq 0. \quad \ldots(22)$$

Now $\frac{\partial TC(1)}{\partial T_0}=0$ implies that

$$c_1[\alpha T_0+\frac{5\alpha\theta T_0^3}{9}-\alpha\mu-\alpha\theta T_0^2\mu+\frac{\alpha\theta\mu^3}{9}]+c_2\left[\left[\alpha T+\gamma\frac{T^2}{2}+\gamma T_0\right]\right.$$
$$\left.-\alpha\mu+\gamma\frac{\mu^2}{2}+\gamma T_0\mu\right]+c\left[\alpha-\alpha\gamma T_0-\frac{5\alpha\gamma\theta T_0^3}{9}+\left(\alpha\gamma+\frac{\alpha\gamma\theta T_0^2}{2}\right)\mu \quad +\frac{\alpha\theta\mu^3}{9}\right]$$
$$+cI_c[\alpha T_0+\frac{5\alpha\theta T_0^2}{12}-\left(\alpha+\frac{\alpha\theta T_0^2}{2}\right)\mu+\frac{\alpha\theta\mu^3}{9}\Big]-sI_e\alpha T_0=0, \quad \ldots(23)$$

and $\frac{\partial TC(1)}{\partial T}=0$ implies that

$$A+c_1[\frac{S}{\alpha(p+1)}[1-\alpha p\mu]^{\frac{p+1}{p}}+\frac{\alpha T_0^2}{2}+\frac{5\alpha\theta T_0^4}{36}-\alpha T_0\mu-\alpha\theta T_0^3\mu$$
$$+\frac{\alpha\mu^2}{2}+\frac{\alpha\theta T_0\mu^3}{9}-\frac{\alpha\theta\mu^4}{12}]+c_2\left[\left[\alpha\frac{T^2}{2}-\gamma T_0\frac{T^2}{2}+\frac{\gamma T^3}{3}\right]\right.$$
$$\left.+\alpha\left(T_0\mu-\frac{\mu^2}{2}\right)-\gamma T_0\frac{\mu^2}{2}-\frac{\gamma}{2}\left(T_0^2\mu+\frac{\mu^3}{3}\right)\right]$$
$$+c\left[S-\alpha T_0-\frac{\alpha\gamma T_0^2}{2}-\frac{5\alpha\gamma\theta T_0^4}{36}+\left(\alpha+\alpha\gamma T_0+\frac{\alpha\gamma\theta T_0^3}{6}\right)\mu\right.$$
$$\left.-\frac{\alpha\gamma\mu^2}{2}+\frac{\alpha\theta T_0\mu^3}{9}-\frac{\alpha\theta\gamma\mu^4}{12}\right]+cI_c\left[\frac{S}{\alpha(p+1)}\left[(1-\alpha pM)^{\frac{p+1}{p}}-(1-\alpha p\mu)^{\frac{p+1}{p}}\right]\right.$$
$$\left.+\frac{\alpha T_0^2}{2}+\frac{5\alpha\theta T_0^3}{36}-\left(\alpha T_0+\frac{\alpha\theta T_0^3}{6}\right)\mu+\frac{\alpha\mu^2}{2}-\frac{\alpha\theta\mu^4}{12}+\frac{\alpha\theta T_0\mu^3}{9}\right]$$
$$-sI_e\left[-\frac{\mu(1-\alpha p\mu)^{\frac{\beta+p}{p}}}{(\beta+p)}+\frac{\left\{1-(1-\alpha p\mu)^{\frac{\beta+2p}{p}}\right\}}{\alpha(\beta+p)(\beta+2p)}+\frac{\alpha}{2}(T_0^2-\mu^2)\right]=0. \quad \ldots(24)$$

Similarly $\frac{\partial TC(2)}{\partial T_0}=0$ implies that

$$c_1[\alpha T_0+\frac{5\alpha\theta T_0^3}{9}-\alpha\mu-3\alpha\theta T_0^2\mu]+c_2\left[\left[\alpha T+\gamma\frac{T^2}{2}+\gamma T_0 T\right]\right.$$
$$\left.-\alpha\mu+\gamma\frac{\mu^2}{2}+\gamma\mu T_0\right]$$
$$+c\left[-\alpha-\alpha\gamma T_0-\frac{5\alpha\gamma\theta T_0^3}{9}+\left(\alpha\gamma+\frac{\alpha\gamma\theta T_0^2}{2}\right)\mu \quad +\frac{\alpha\theta\mu^3}{9}\right]$$
$$+cI_c\left[\alpha T_0+\frac{5\alpha\theta T_0^2}{18}-\left(\alpha+\frac{\alpha\theta T_0^2}{2}\right)\mu+\frac{\alpha\theta\mu^3}{9}\right]-sI_e\alpha T_0, \quad \ldots(25)$$

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and $\frac{\partial TC(2)}{\partial T} = 0$ implies that

$$A + c_1[\frac{S}{\alpha(p+1)}[1-\alpha p\mu]^{\frac{p+1}{p}} + \frac{\alpha T_0^2}{2} + \frac{5\alpha\theta T_0^4}{36} - \alpha T_0\mu - \alpha\theta T_0^3\mu$$
$$+\frac{\alpha\mu^2}{2} + \frac{\alpha\theta T_0\mu^3}{9} - \frac{\alpha\theta\mu^4}{12}] + c_2\left[\left[\alpha\frac{T^2}{2} - \gamma T_0\frac{T^2}{2} + \frac{\gamma T^3}{3}\right]\right.$$
$$\left.-\alpha\left(T_0\mu - \frac{\mu^2}{2}\right) + \gamma T_0\frac{\mu^2}{2} + \frac{\gamma}{2}\left(T_0^2\mu + \frac{\mu^3}{3}\right)\right]$$
$$+c\left[S - \alpha T_0 - \frac{\alpha\gamma T_0^2}{2} - \frac{5\alpha\gamma\theta T_0^4}{36} + \left(\alpha + \alpha\gamma T_0 + \frac{\alpha\gamma\theta T_0^3}{6}\right)\mu\right.$$
$$\left.-\frac{\alpha\gamma\mu^2}{2} + \frac{\alpha\theta T_0\mu^3}{9} - \frac{\alpha\theta\gamma\mu^4}{12}\right]$$
$$+cI_c\left[\frac{\alpha T_0^2}{2} + \frac{5\alpha\theta T_0^3}{36} - \left(\alpha T_0 + \frac{\alpha\theta T_0^3}{6}\right)\mu + \frac{\alpha\mu^2}{2} - \frac{\alpha\theta\mu^4}{12} + \frac{\alpha\theta T_0\mu^3}{9}\right]$$
$$-sI_e\left[-\frac{\mu(1-\alpha p\mu)^{\frac{\beta+p}{p}}}{(\beta+p)} + \frac{\left\{1-(1-\alpha p\mu)^{\frac{\beta+2p}{p}}\right\}}{\alpha(\beta+p)(\beta+2p)} + \frac{\alpha}{2}(T_0^2 - \mu^2)\right] = 0. \qquad \ldots(26)$$

Similarly $\frac{\partial TC(3)}{\partial T_0} = 0$ implies that

$$c_1[\alpha T_0 + \frac{5\alpha\theta T_0^3}{9} - \alpha\mu - 3\alpha\theta T_0^2\mu]$$
$$+c_2\left[\left[\alpha T + \gamma\frac{T^2}{2} + \gamma T_0 T\right] - \alpha\mu + \gamma\frac{\mu^2}{2} + \gamma\mu T_0\right]$$
$$+c\left[-\alpha - \alpha\gamma T_0 - \frac{5\alpha\gamma\theta T_0^3}{9} + \left(\alpha\gamma + \frac{\alpha\gamma\theta T_0^2}{2}\right)\mu + \frac{\alpha\theta\mu^3}{9}\right]$$
$$-sI_e[\alpha T_0 + (M-T_0)\left\{(\alpha+\gamma T_0)(M+T_0) - \frac{2\gamma}{3}(M^2 + MT_0 + T_0^2)\right\}$$
$$+(M-T_0)^2\left\{(\alpha+\gamma T_0) + \frac{\gamma}{3}(2M+T_0)\right\}\Big] = 0, \qquad \ldots(27)$$

and $\frac{\partial TC(3)}{\partial T} = 0$ implies that

$$A + c_1[\frac{S}{\alpha(p+1)}[1-\alpha p\mu]^{\frac{p+1}{p}} + \frac{\alpha T_0^2}{2} + \frac{5\alpha\theta T_0^4}{36} - \alpha T_0\mu - \alpha\theta T_0^3\mu$$
$$+\frac{\alpha\mu^2}{2} + \frac{\alpha\theta T_0\mu^3}{9} - \frac{\alpha\theta\mu^4}{12}] + c_2\left[\left[\alpha\frac{T^2}{2} - \gamma T_0\frac{T^2}{2} + \frac{\gamma T^3}{3}\right]\right.$$
$$\left.-\alpha\left(T_0\mu - \frac{\mu^2}{2}\right) + \gamma T_0\frac{\mu^2}{2} + \frac{\gamma}{2}\left(T_0^2\mu + \frac{\mu^3}{3}\right)\right]$$
$$+c\left[S - \alpha T_0 - \frac{\alpha\gamma T_0^2}{2} - \frac{5\alpha\gamma\theta T_0^4}{36} + \left(\alpha + \alpha\gamma T_0 + \frac{\alpha\gamma\theta T_0^3}{6}\right)\mu\right.$$
$$\left.-\frac{\alpha\gamma\mu^2}{2} + \frac{\alpha\theta T_0\mu^3}{9} - \frac{\alpha\theta\gamma\mu^4}{12}\right]$$

$$-sI_e\alpha T_0 = 0. \quad \ldots(28)$$

Similarly the values of second derivatives can be obtained. Solving equations (23), (25) and (27) we can obtain the values of $T_0^*(1)$, $T_0^*(2)$ and $T_0^*(3)$ respectively. Substituting these values in (24), (26) and (28), we can obtain the values of $T^*(1)$, $T^*(2)$ and $T^*(3)$ respectively. Now $T_0^*(1)$ and $T^*(1)$ are the optimal values of T_0 and T if $0 < M \leq \mu$ and the second derivatives satisfy (22). $T_0^*(2)$ and $T^*(2)$ are the optimal values of T_0 and T if $\mu < M \leq T_0$ and second derivatives satisfy (22). $T_0^*(3)$ and $T^*(3)$ are the optimal values of T_0 and T if $M \geq T_0$ and second derivatives satisfy (22). Thus we get the optimal values of T_0^* and T^*. Putting these values in equation (8) and (12) we can obtain the values of S^* and Q^*. Mathematika or matlab softwares can be used for illustration.

Concluding Remarks:

In this paper, an inventory model has been developed for deteriorating products with life time under trade credit. Three components demand rate has been considered. During life time, the demand rate is stock dependent. During deterioration period, the demand rate is constant. During shortage period the demand rate is time dependent. Deterioration rate has been taken linear function of t. Permissible delay in payments is allowed which has been considered in three cases. This model can further be developed for other forms of demand rate and power pattern form of deterioration.

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