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Effect of Chemical Reaction on Unsteady Laminar Mhd Free Convective Flow of A Micropolar Fluid Past an Infinite Plate Through Porous Medium

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Abstract:
The main objective of this paper is to study of the flow of micro polar fluids due to Eringen, and the effect of chemical reaction on unsteady laminar MHD free convection flow of micro polar fluid past an infinite vertical isothermal plate through porous medium also it has been studied numerically. It is assumed that there is no anti symmetric part of the stress on the boundary. The governing nonlinear partial differential equations are solved by finite difference method. The results are obtained for velocity, micro rotation, temperature and concentration profiles. The numerical results are presented graphically for different values of the parameters entering into the problem. Finally, the numerical values of skin friction are presented in pictorial form.

Keyword: - MHD, unsteady, free convection, heat and mass transfer, micropolar, porous medium

1. Introduction

The flow of an incompressible viscous fluid past an impulsively started infinite horizontal plate, in its own plane, was first studied by Stokes [6]. It is also known as Ravleigh's problem in the literature. Following Stokes' analysis, Soundalgekar [4] first presented free convection effects on the Stokes problem for an infinite vertical plate. Soundalgekar [5] has studied mass transfer effects on flow past an impulsively started infinite vertical plate.
concentration buoyancy effects. The transversely applied magnetic field and magnetic Reynolds number are very small and hence, the induced magnetic field is negligible [11].

The $x'-$axis taken along the plate in the vertical upward direction and the $y'-$axis is taken normal to the plate. At time t' >0, the plate temperature is raised to T'_w causing the creation of free convection currents due to temperature difference $T'_w - T'_\infty$. With this assumption the flow variables are functions of y' and t' only. Then under usual Boussinesq's approximation, the unsteady free convection flow of a micro polar fluid can be shown to be governed by the following system of equations:

$$\rho\frac{\partial u'}{\partial t'} = (\mu+\kappa)\frac{\partial^2 u'}{\partial y'^2} + \kappa\frac{\partial \omega'}{\partial y'}$$

$$+\rho g\beta(T'-T'_\infty) + \rho g\beta^*(C'-C'_\infty) - \frac{\sigma B_0^2}{\rho}u' - \frac{\upsilon}{K'}u' \quad \ldots(1)$$

$$\rho j\frac{\partial \omega'}{\partial t'} = \gamma\frac{\partial^2 \omega'}{\partial y'^2} - 2\left(\omega' + \frac{1}{2}\frac{\partial u'}{\partial y'}\right) \quad \ldots(2)$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} \quad \ldots(3)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - S(C'-C_\infty) \quad \ldots(4)$$

The initial and boundary conditions are:

$$\left.\begin{array}{llllll} t'\le 0: & u'=0, & T'=T'_\infty, & C'=C'_\infty, & \omega'=0 & \text{for all } y \\ t'>0: & u'=0, & T'=T'_w, & C'=C'_w, & \omega'=-\frac{1}{2}\frac{\partial u'}{\partial y'} & \text{at } y=0 \\ & u'=0, & T'=T'_\infty, & C'=C'_\infty, & \omega'=0 & \text{as } y\to\infty \end{array}\right\} \quad \ldots(5)$$

Here ρ is the density, u' is the velocity of the micro polar fluid in the $x'-$direction, μ is the coefficient of viscosity, κ is the vortex viscosity, ω' is the component of microrotation, g is

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the acceleration due to gravity, γ is the spin-gradient viscosity, c_p is the specific heat at constant pressure, κ is the thermal conductivity, K' is the porosity parameter, D is the chemical molecular diffusivity, T' and C' are the temperature and concentration of the fluid, B_0^2 is the applied magnetic field. Also, the boundary condition $\omega'(0) = -\frac{1}{2}\left(\frac{\partial u}{\partial y}\right)_{y=0}$ corresponds to no anti symmetric part of the stress on the boundary, S is chemical reaction parameter (non dimensional), γ is chemical reaction parameter.

On introducing the following non-dimensional quantities:

$$U_0 = \frac{L}{\upsilon}, \quad u = \frac{u'}{U_0}, \quad y = \frac{y'}{L}, \quad t = \frac{\upsilon t'}{L^2}, \quad \omega = \frac{\omega' L}{U_0}$$

$$\Pr = \frac{\mu C_p}{k}, \quad Sc = \frac{\upsilon}{D}, \quad \eta_1 = \frac{\kappa}{\rho\upsilon}, \quad \eta_2 = \frac{\gamma}{\rho j \upsilon}, \quad \eta_3 = \frac{\upsilon^2}{jU_0^2}$$

$$M = \frac{\sigma B_0^2 L^2}{\rho\mu}, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad \phi = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad K' = \frac{L^2 K}{U_0 \rho}$$

$$Gr = \frac{\upsilon g\beta\left(T'_w - T'_\infty\right)}{U_0^3}, \quad Gc = \frac{\upsilon g\beta^*\left(C'_w - C'_\infty\right)}{U_0^3}, \quad \mu = \upsilon\rho, \; \gamma = SL^2/\upsilon$$

In equations (1) to (5), we have

$$\frac{\partial u}{\partial t} = (1+\eta_1)\frac{\partial^2 u}{\partial y^2} + \eta_1\frac{\partial \omega}{\partial y} + Gr\theta + Gc\phi - \left(M + \frac{1}{K}\right)u \qquad ...(6)$$

$$\frac{\partial \omega}{\partial t} = \eta_2\frac{\partial^2 \omega}{\partial y^2} - 2\eta_1\eta_3\left(\omega + \frac{1}{2}\frac{\partial u}{\partial y}\right) \qquad ...(7)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} \qquad ...(8)$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} - \gamma\phi \qquad ...(9)$$

With following initial and boundary conditions:

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$$\left.\begin{array}{ll} t \le 0: & u = 0, \quad \theta = 0, \quad \phi = 0, \quad \omega = 0 \quad \text{for all} \quad y \\ t > 0: & u = 0, \quad \theta = 1, \quad \phi = 1, \quad \omega = -\frac{1}{2}\frac{du}{dy} \quad at \quad y = 0 \\ & u = 0, \quad \theta = 0, \quad \phi = 0, \quad \omega = 0 \quad as \quad y \to \infty \end{array}\right\} \quad ...(10)$$

It is very difficult to drive analytic solutions to equations (6) to (9), satisfying the conditions (10). We now drive the finite – difference solutions. Then the explicit finite – difference scheme for these equation are as follows;

$$\left(\frac{u_{i,j+1} - u_{i,j}}{\Delta t}\right) = (1+\eta_1)\left[\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{(\Delta y)^2}\right] + \eta_1\left(\frac{\omega_{i,j+1} - \omega_{i,j}}{\Delta y}\right)$$

$$+ Gr\theta_{i,j} + Gc\phi_{i,j} - \left(M + \frac{1}{K}\right)u_{i,j} \quad \ldots(11)$$

$$\left(\frac{\omega_{i,j+1} - \omega_{i,j}}{\Delta t}\right) = \eta_2\left[\frac{\omega_{i+1,j} - 2\omega_{i,j} + \omega_{i-1,j}}{(\Delta y)^2}\right] - 2\eta_1\eta_3\left[\omega_{i,j} + \frac{1}{2}\frac{(u_{i+1,j} - u_{i,j})}{\Delta y}\right] \quad ...(12)$$

$$\left(\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t}\right) = \frac{1}{\Pr}\left[\frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{(\Delta y)^2}\right] \quad ...(13)$$

$$\left(\frac{\phi_{i,j+1} - \phi_{i,j}}{\Delta t}\right) = \frac{1}{Sc}\left[\frac{\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}}{(\Delta y)^2}\right] - \gamma\phi_{i,j} \quad ...(14)$$

And the initial and boundary conditions are:

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$$\left.\begin{array}{llllr} u(i,0)=0, & \theta(i,0)=0, & \phi(i,0)=0, & \omega(i,0)=0, & i>0 \\ u(0,j)=0, & \theta(0,j)=1, & \phi(0,j)=1, & \omega(0,j)=-\dfrac{1}{2}\dfrac{u(1,j)}{\Delta y}, & j>0 \\ u(82,j)=0, & \theta(82,j)=0, & \phi(82,j)=0, & \omega(82,j)=0, & j>0 \end{array}\right\} \quad ...(15)$$

Here i refers to y and j refers to time t. The mesh system is divided by taking, $\Delta y = 0.1$. The infinity is assumed at $y \sim 4.1$ and $\Delta y = 0.1$, $\Delta t = 0.0005$. The computation is carried out and the numerical values of u, □, □ and □ are computed in the usual manner. To judge the accuracy of the convergence of the finite difference scheme, the same program was run with smaller values of Δt, i.e., Δt = 0.0009, 0.001 and no significant change was observed. Hence, we conclude that the finite difference scheme is stable and convergent. It is now necessary to study the effects of material constants η_1, η_2 and η_3 on the flow of micropolar fluid and hence we have chosen η_1, η_2, η_3 all positive. The parameters η_1 depends upon the shape and concentration of the microelements and therefore η_1 measures the concentration of the microelements in the micropolar fluid. The other two parameters η_2 and η_3 depends upon the fluid properties such as the relative size of microstructure in relation to a geometrical length.

We now study the skin friction. It is given by

$$\tau = -\left(1+\frac{1}{2}\eta_1\right)\left(\frac{du}{dy}\right)_{y=0} \quad ...(16)$$

The numerical values of $-\left(\dfrac{du}{dy}\right)_{y=0}$ are computed by using the Newton's eleven point formula.

2. Results and Discussion

Numerical calculations have been carried out using finite difference for dimensionless velocity, micro rotation, temperature and concentration profiles for different values of parameters and are displayed in Figures-(1) to (18).

In figures – (1) to (4) the velocity, micro rotation concentration,and temperature

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profile laminar flow is plotted again span wise coordinate y for Gr = 2, Gc = 3, M = 0.01, K = 2, Sc = 0.4, t = 0.04 and material constants η_1 = 0.2, η_2 = 0.3, η_3 = 0.4 and different values of chemical reaction parameter γ . The results shows that increasing value of chemical reaction parameter γ, results in increasing velocity ,micro rotation and concentration profile of laminar flow,but there is no effect of chemical reaction parameter on the temperature.

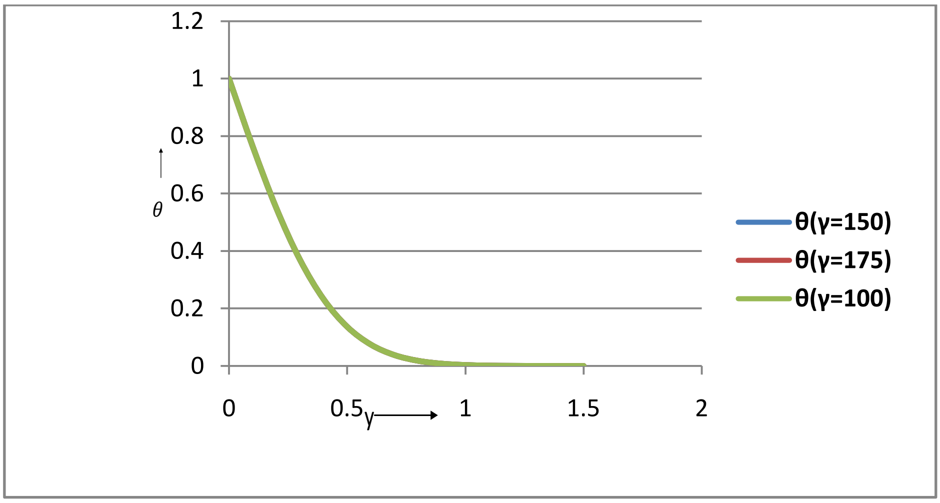

Figure : 1

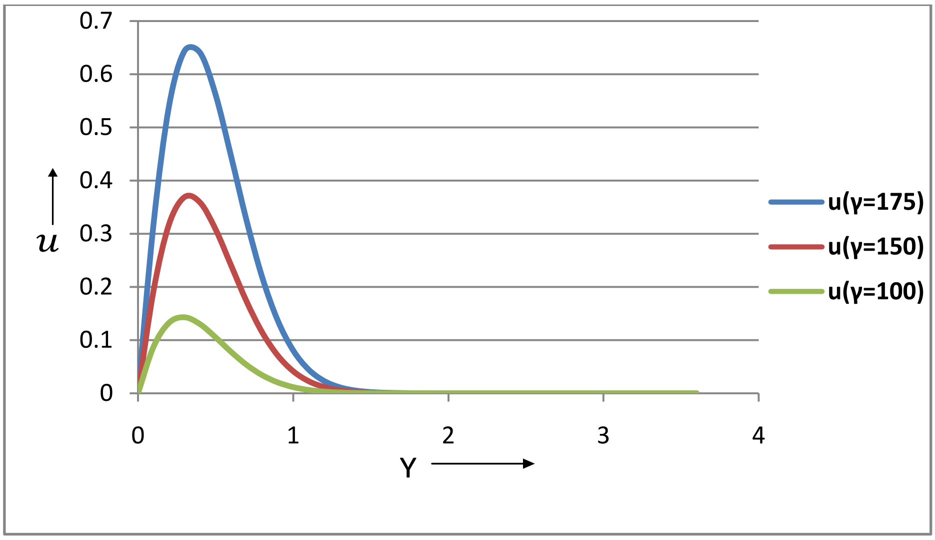

Figure :2

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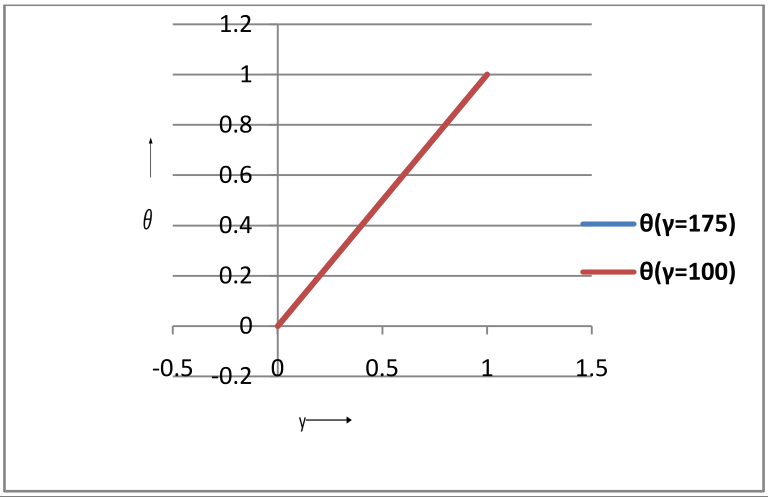

Figure : 3

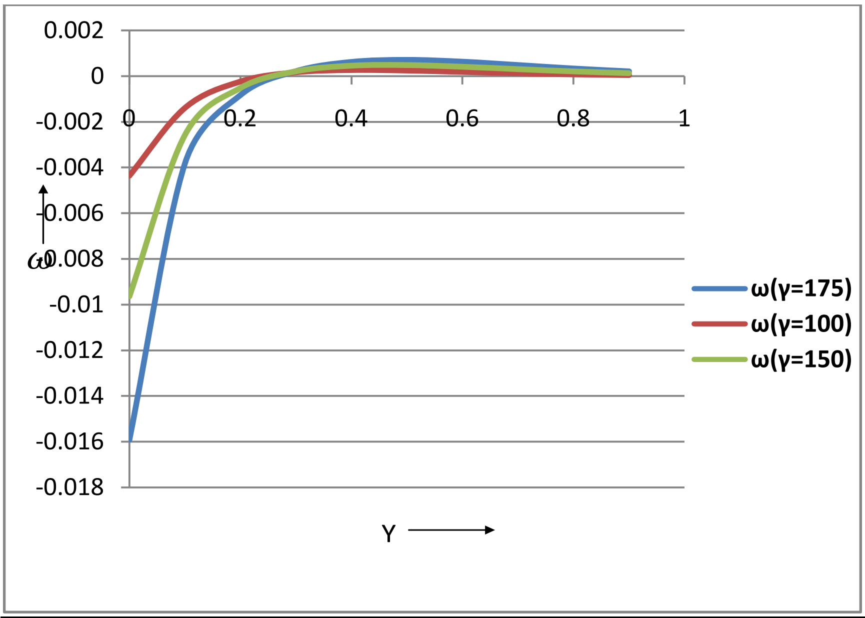

Figure : 4

3. Conclusion

From the graph it is observed that

(1) Velocity of fluid increases as we increase chemical reaction.

(2) Concentration increases as we increase chemical reaction.

(3) Micro rotation increases as we increase chemical reaction.

(4) Temperature is unaffected by chemical reaction.

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