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Interacting and Non-interacting Two-fluid Models for Dark Energy in $f(R)$ Theory

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Abstract:
The paper is devoted to study the evolution of the dark energy parameter within the scope of a spatially flat and isotropic Friedmann –Robertson-Walker (FRW) model filled with barotropic fluid and dark energy in $f(R)$ gravity. The model of $f(R)$ i.e. $f(R) = R + bR^m$ has been studied, for the solution of the field equations. The physical and kinematical features of the models are studied and discussed..

Keywords: Dark energy, FRW Universe, $f(R)$ theory

1. Introduction:

Among the various modifications of the general theory of relativity (GR), the $f(R)$ theory of gravity is treated most seriously during the last decade. In this theory, a general function of Ricci scalar is used instead of standard Einstein-Hilbert Lagrangian R. The $f(R)$ theory of gravity is considered most suitable due to cosmologically important $f(R)$ models. These models consist of higher order curvature invariants as functions of the Ricci scalar. Viable $f(R)$ gravity models (Nojiri and Odintsov [1]) have been proposed which show the unification of early time inflation and late-time acceleration. However, the field equations of $f(R)$ gravity models turn out to be fourth order differential equations in the metric formalism and therefore they are difficult to analyze. By choosing suitable functional form of $f(R)$, one can describe accelerated expansion of the Universe because the additional degree of freedom of the function $f(R)$ plays a role of a scalar field, which is called scalaron, and is responsible for the acceleration. Weyl [2] and Eddington [3] were the first who studied the action in the context of $f(R)$ theory of gravity. Jakubiec and Kijowski [4] investigated theories of gravitation with non-linear Lagrangian. The most spaciously discovered exact solutions in $f(R)$ gravity are the spherically symmetric solutions which were found by Multamaki and Vilja [5]. On the other hand, models containing inverse powers of R were also analyzed (Capozziello et al. [6], Nojiri and Odintsov [7-8]) which help to explain the current accelerated expansion of the universe but faced some problems (Chiba [9]; Dominguez and Barraco [10]). $f(R)$ theories can pass the Solar System tests have been shown by Nojiri and Odintsov [11-13] and Faraoni [14]. Chiba et al [15] have shown that there exists a mathematically equivalent between $f(R)$ gravity and scalar-tensor theory of gravity. Cylindrical symmetric solutions in $f(R)$ theory were explored by Momeni, Azadi and Nouri-Zonoz [16]. This work was extended by Momeni and Gholizade [17] to the general cylindrical symmetric solutions. Plane symmetric static solution and vacuum solutions of Bianchi types I and V spacetimes in $f(R)$ theory of gravity by using metric affine approach were studied by Sharif and Farasat [18-20] . Bianchi type I,III and Kantowski-Sachs spacetimes in $f(R)$ gravity were studied by Farasat [21-22]. Capozziello and De Laurentis [23] reviewed the most important aspects of the extended theories of gravity. Shojai and Shojai [24] have obtained some new static spherically symmetric interior solutions which are physically acceptable for a star in metric $f(R)$ gravity. M.Amir and S.Sattar [25] studied Locally Rotationally Symmetric Vacuum Solutions in $f(R)$ gravity. M.Sharif and Z.Zahra [26] study static spherically symmetric wormhole solutions in $f(R)$ gravity. M.Amir and S.Naheed [27] explored the spatially homogeneous rotating solution in $f(R)$ theory of gravity. H.Kausar and I. Noureen [28] determined the electromagnetic field impressions on the instability of spherically symmetric collapsing compact object in $f(R)$ framework. Recently Katore and Shaikh [29] studied Bianchi Type III Cosmological Models with Bulk Viscosity in $f(R)$ gravity .

An interacting two-fluid scenario for quintom dark energy was investigated by Xin [30]. The interacting models of Dark Energy (DE) in a different context have been discussed by Setare [31-33] and Setare & Saridakis [34]. Liang et al.[35] investigated the cosmological evolution of a two-field dilation model of dark energy. Setare et al.[36] studied the viscous dark tachyon cosmology in interacting and non-interacting cases in non-flat FRW Universe. Sheykhi & Setare [37]studied new interacting agegraphic viscous Dark Energy with varying G .The evolutions of the dark energy parameter within the framework of an FRW cosmological model filled with two fluids have been studied by Hassan et al [38]. Amirhashchi et al.[39-40], Pradhan et al. [41], and Saha et al. [42] have studied the two-fluid scenario for DE in an FRW Universe in a different context. Very recently, Singh & Chaubey [43] examined interacting DE in Bianchi type I space-time.

Motivated by the above researchers work, in this paper, the evolution of the dark parameter within the scope of a spatially flat and isotropic Friedmann – Robertson-Walker (FRW) model filled with barotropic fluid and dark energy in the framework of $f(R)$ gravity has been discussed.

2. $f(R)$ Gravity Formalism:

The $f(R)$ theory of gravity is the generalization of General Relativity. The three main approaches in $f(R)$ theory of gravity are "Metric Approach", "Palatini formalism" and "affine $f(R)$ gravity". In metric approach, the connection is the Levi-Civita connection and variation of the action is done with respect to the metric tensor. While, in Palatini formalism, the metric and the connection are independent of each other and variation is done for the two mentioned parameters independently. In metric-affine $f(R)$ gravity, both the metric tensor and connection are treating independently and assuming the matter action depends on the connection as well.

The action for this theory is given by

$$S=\frac{1}{2k^2}\int d^4\sqrt{-g}\,f(R)+\int d^4xL_m(g_{\mu\nu},\psi_m). \tag{1}$$

Here $f(R)$ is a general function of the Ricci Scalar , $k^2=8\pi G=1$, g is the determinant of the metric $g_{\mu\nu}$ and L_m is the metric Lagrangian that depends on $g_{\mu\nu}$ and the matter field ψ_m. It is noted that this action is obtained just by replacing R by $f(R)$ in the standard Einstein – Hilbert action.

The corresponding field equations are found by varying the action with respect to the metric $g_{\mu\nu}$

$$F(R)\,R_{\mu\nu}-\frac{1}{2}f(R)g_{\mu\nu}-\nabla_\mu\nabla_\nu F(R)+g_{\mu\nu}\square F(R)=T^M_{\mu\nu}, \tag{2}$$

where $\square\equiv\nabla^\mu\nabla_\mu$, $F(R)\equiv\dfrac{df(R)}{dR}$, (3)

∇_μ is the covariant derivative and $T_{\mu\nu}$ is the standard matter energy-momentum tensor derived from the Lagrangian L_m. These are fourth order partial differential equations in the metric tensor due to the last two terms on the left hand side of the equation. Here the energy momentum is two fluid energy-momentum tensor consisting of a dark field and barotropic fluid .

3. Metric and Field Equations:

The adequacy of spatially homogeneous and isotropic FRW models for describing the present state of the universe is basis for expecting that it is equally suitable for describing its early stages of evolution. The spatial homogenous and isotropic flat Friedmann-Robertson-Walker (FRW) metric is of the form

$$ds^2 = -dt^2 + a^2(t)dx^2, \quad (4)$$

where $a(t)$ being the scale factor of the universe.

Granda and Olivers [44] studied the correspondence of new holographic dark energy in flat FRW universe. F. Darabi [45] studied the spatially flat FRW metric for the reconstruction of $f(R)$, $f(T)$ and $f(G)$ models inspired by variable deceleration parameter. H.Saadat and B.Pourhassan [46] studied FRW bulk viscous cosmology with modified Chaplygin gas in flat space. Y.D. Xu and Z.G. Huang [47] studied a cosmological model with the sign-changeable interaction between variable generalized Chaplygin gas (VGCG) and dark matter by considering spatially flat FRW universe. Singh et al [48] presented an isotropic and homogeneous flat FRW cosmological model for bulk viscous fluid distribution.

The corresponding Ricci Scalar curvature for flat FRW model is given by

$$R = 6\left[\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right], \quad (5)$$

where over head dot represents derivative with respect to t.

In a co-moving coordinate system, the field equations (2) for the line element (4) lead to

$$\left(2\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a}\right)F - \frac{1}{2}f(R) - 2\frac{\dot{a}}{a}\dot{F} - \ddot{F} = p_{tot}, \quad (6)$$

$$-3\frac{\ddot{a}}{a}F + \frac{1}{2}f(R) + 3\frac{\dot{a}}{a}\dot{F} = \rho_{tot}. \quad (7)$$

The overhead dot represents the differentiation with respect to time t and $p_{tot} = (p_m + p_D)$, $\rho_{tot} = (\rho_m + \rho_D)$. Here p_m and ρ_m are pressure and energy density of barotropic fluid, p_D and ρ_D are pressure and energy density of dark fluid respectively.

The Bianchi identity $G_{ij}^{;j} = 0$ leads to $T_{ij}^{;j} = 0$ which yields

$$\dot{\rho}_{tot} + 3\frac{\dot{a}}{a}(\rho_{tot} + p_{tot}) = 0. \quad (8)$$

The equation of state (EoS) of the barotropic fluid and dark fluid are given by

$$w_m = \frac{p_m}{\rho_m}, \tag{9}$$

$$w_D = \frac{p_D}{\rho_D}. \tag{10}$$

The physical quantities of observational interest in cosmology are spatial volume V, mean Hubble parameter H, the expansion scalar θ , the mean anisotropy parameter A_m , the shear scalar σ^2 and the deceleration parameter q . They are defined as

Spatial Volume $V = a^3$. (11)

Mean Hubble Parameter $H = \left(\frac{\dot{a}}{a}\right)$. (12)

Scalar Expansion $\theta = 3H$. (13)

Anisotropic Parameter $A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2$. (14)

where H_i (i = 1, 2, 3) are the directional Hubble parameters in the directions of x, y and z axes respectively.

Shear Scalar $\sigma^2 = \frac{3}{2}A_m H^2$. (15)

DecelerationParameter $q = -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{\ddot{a}}{aH^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1$. (16)

The Hubble parameter H and deceleration parameter q are some of the most important observational quantities in cosmology, for any physically relevant model. The first quantity sets the present time scale of the evolution while the second one is telling us that present stage is speeding up instead of slowing down as expected before the Supernovae type Ia observations. Here the deceleration parameter q measures the rate of expansion of the universe. The sign of q indicates the state of expanding universe. If $q < 0$, then it represents inflation. Also if $q > 0$, then it represents deflation of the universe while $q = 0$ shows expansion with constant velocity. Recent observations (Riess et al. [49-51]; Perlmutter et al.[52]) have suggested that the rate of expansion of the universe is currently accelerating, perhaps due to dark energy. This yields negative values of the Deceleration Parameter.

In the following sections we deal with two cases

(i) Non-interacting two-fluid model and
(ii) Interacting two-fluid model.

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4. Non-interacting two fluid models

Firstly the two-fluids do not interact with each other. Therefore, the general form of conservation equation (8) leads us to write the conservation equation for the dark and barotropic fluid separately.

The energy conservation equation $T^{\mu\nu}_{;\nu}=0$ of the perfect fluid leads to

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}(\rho_m + p_m) = 0, \tag{17}$$

whereas the energy conservation equation $T^{\mu\nu}_{;\nu}=0$ of the dark energy component yields

$$\dot{\rho}_D + 3\frac{\dot{a}}{a}(\rho_D + p_D) = 0. \tag{18}$$

Integration of equation (17) leads to

$$\rho_m = \rho_0 a^{-3(1+\omega_m)}, \tag{19}$$

where ρ_o is an integrating constant.

By using equation (19) in equations (6) and (7), we first obtain the ρ_D and p_D in term of scale factor $a(t)$

$$\rho_D = -3\frac{\ddot{a}}{a}F + \frac{1}{2}f(R) + 3\frac{\dot{a}}{a}\dot{F} - \rho_0 a^{-3(1+\omega_m)} \tag{20}$$

$$p_D = \left[2\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a}\right]F - \frac{1}{2}f(R) - 2\frac{\dot{a}}{a}\dot{F} - \ddot{F} - \rho_0\omega_m a^{-3(1+\omega_m)}. \tag{21}$$

Since, the field equations are coupled system of highly non-linear differential equations and seeking physically realistic solutions to the field equations for applications in cosmology and astrophysics. Let us assume that the scale factor $a(t)$ to be power law function of time as

$$a(t) = n_0 t^n, \tag{22}$$

where n_0 is a positive constant and n is a positive real number which determines the expansion of scale factor in different phases of the evolution of the universe .This model supports cosmic acceleration expansion for $n>1$.The motivation to choose such scale factor is behind the fact that the universe is accelerated expansion at present and decelerated expansion in the past. Also, the transition redshift from deceleration expansion to accelerated expansion is about 0.5. For $n=1$determines the marginal inflation $a(t)\propto t$, $n=\frac{1}{2}$ for radiation dominated phase $a(t)\propto t^{\frac{1}{2}}$, $n=\frac{2}{3}$ for matter dominated phase $a(t)\propto t^{\frac{2}{3}}$ and $n=\frac{4}{3}$ determines the accelerated phase

$a(t) \propto t^{\frac{4}{3}}$ of the universe. Therefore, the power law solution (20) describes the expansion of the scale factor in decelerating and accelerating universe.

It is not simple to determine analytic solution of the scale factor of the universe with cosmic time in closed functional form. Hence adopt numerical technique to study the behavior of the cosmological models based on the parameters of the modified gravitational action. For simplicity consideration of $f(R)$ of two different forms in the next sections.

4.1. Model:

Firstly the super gravity inspired model (Noakes [53],M.Sharif and S.Arif [54]) has been considered.

Let $f(R) = R + bR^m$, (23)

where b and m are constants. It is also called the inflation model realized by the terms R^2 for m=2.The model has a stability criterion which is bounded to $b > 0$, i.e. $f''(R) > 0$.Einstein's theory is retrieved if $b = 0$. In $f(R)$ gravity, all these characteristics are also observed, thus the stability condition for this theory takes the form $f''(R) > 0$ (Sortiriou and Faraoni [55]).The configuration of second order derivative decides whether the model is viable or not. Any $f(R)$ is supposed to be suitable in GR and Newtonian limits if $f''(R) > 0$. For our proposed form of $f(R)$, $m > 2$ and b is a positive real number, in order to fulfill stability criterion and demonstrate accelerated expansion of the universe.

Using equations (5) and (22) we obtain the corresponding Ricci Scalar curvature as

$$R = \frac{12n^2 - 6n}{t^2}. \tag{24}$$

Using equations (23) and (24), we obtain

$$f(R) = \left(\frac{12n^2 - 6n}{t^2}\right) + b\left(\frac{12n^2 - 6n}{t^2}\right)^m. \tag{25}$$

Using equations (3), (23) and (24), we get

$$F(R) \approx bm\left[\frac{12n^2 - 6n}{t^2}\right]^{m-1}. \tag{26}$$

Using equations (20),(22),(24), (25) and(26) , we have

$$\rho_D=\left\{\begin{array}{l}-\dfrac{3(n^2-n)}{t^2}\left[bm\left(\dfrac{12n^2-6n}{t^2}\right)^{m-1}\right]+\dfrac{1}{2}\left[\left(\dfrac{12n^2-6n}{t^2}\right)+b\left(\dfrac{12n^2-6n}{t^2}\right)^{m}\right]\\ +3\dfrac{n}{t}\left[bm(m-1)\left(\dfrac{12n^2-6n}{t^2}\right)^{m-2}\left(\dfrac{12n-24n^2}{t^3}\right)\right]-\rho_0(n_0t^n)^{-3(1+\omega_m)}\end{array}\right\}. \qquad (27)$$

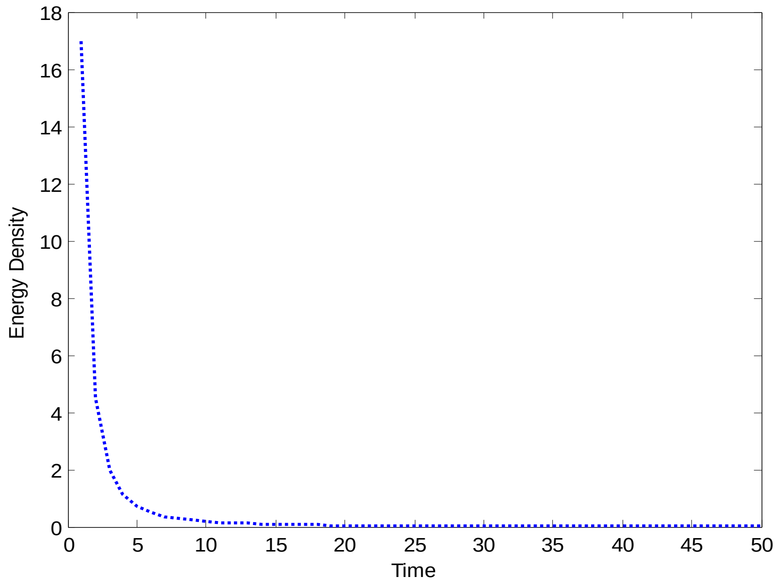

Figure 1 The plot of energy density vs t.

The behavior of ρ_D in terms of cosmic time t is shown in Fig. 1. It is positive decreasing function of time and converges to zero for sufficiently large times. Using equations (21),(22),(24), (25) and(26) , we have

$$p_D=-\left\{\begin{array}{l}-\left[\dfrac{3n^2-n}{t^2}\right]\left[bm\left(\dfrac{12n^2-6n}{t^2}\right)^{m-1}\right]+\dfrac{1}{2}\left[\left(\dfrac{12n^2-6n}{t^2}\right)+b\left(\dfrac{12n^2-6n}{t^2}\right)^{m}\right]\\ +2\dfrac{n}{t}\left[bm(m-1)\left(\dfrac{12n^2-6n}{t^2}\right)^{m-2}\left(\dfrac{12n-24n^2}{t^3}\right)\right]+\\ bm(m-1)\left[(m-2)\left(\dfrac{12n^2-6n}{t^2}\right)^{m-3}\left(\dfrac{12n-24n^2}{t^3}\right)^{2}-\left(\dfrac{12n^2-6n}{t^2}\right)^{m-2}\left(\dfrac{72n^2-36n}{t^4}\right)\right]\\ +\rho_0\omega_m(n_0t^n)^{-3(1+\omega_m)}\end{array}\right\} \qquad (28)$$

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By using equations (27) and (28) in equation (10), we find the equation of state of dark field in term of time as

$$\omega_D = -\frac{\left\{\begin{array}{l} -\left[\frac{3n^2-n}{t^2}\right]\left[bm\left(\frac{12n^2-6n}{t^2}\right)^{m-1}\right]+\frac{1}{2}\left[\left(\frac{12n^2-6n}{t^2}\right)+b\left(\frac{12n^2-6n}{t^2}\right)^{m}\right] \\ +2\frac{n}{t}\left[bm(m-1)\left(\frac{12n^2-6n}{t^2}\right)^{m-2}\left(\frac{12n-24n^2}{t^3}\right)\right]+ \\ bm(m-1)\left[(m-2)\left(\frac{12n^2-6n}{t^2}\right)^{m-3}\left(\frac{12n-24n^2}{t^3}\right)^2-\left(\frac{12n^2-6n}{t^2}\right)^{m-2}\left(\frac{72n^2-36n}{t^4}\right)\right] \\ +\rho_0\omega_m(n_0t^n)^{-3(1+\omega_m)} \end{array}\right\}}{\left\{\begin{array}{l} -\frac{3(n^2-n)}{t^2}\left[bm\left(\frac{12n^2-6n}{t^2}\right)^{m-1}\right]+\frac{1}{2}\left[\left(\frac{12n^2-6n}{t^2}\right)+b\left(\frac{12n^2-6n}{t^2}\right)^{m}\right] \\ +3\frac{n}{t}\left[bm(m-1)\left(\frac{12n^2-6n}{t^2}\right)^{m-2}\left(\frac{12n-24n^2}{t^3}\right)\right]-\rho_0(n_0t^n)^{-3(1+\omega_m)} \end{array}\right\}} \tag{29}$$

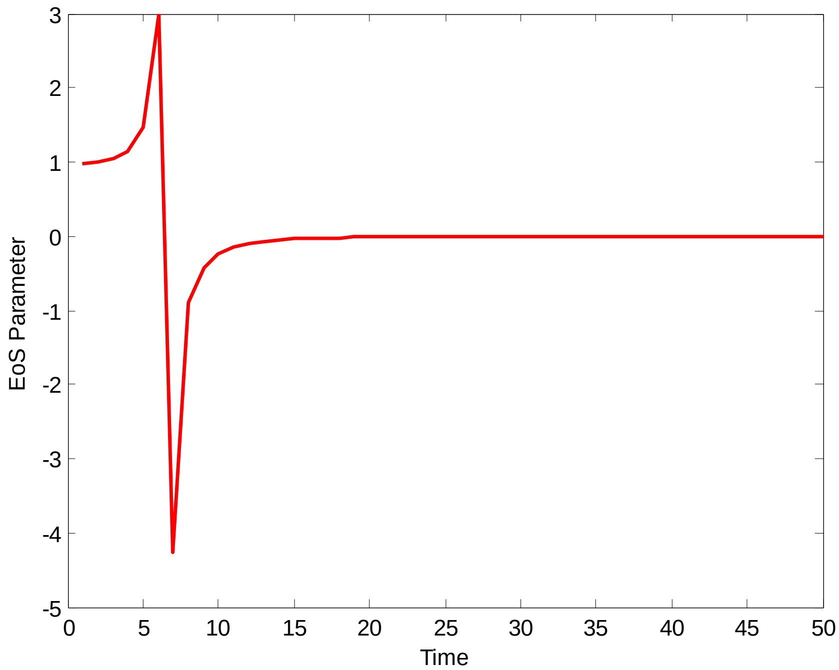

Figure 2 The plot of EoS parameter vs *t*.

The behavior of EoS for dark energy in term of cosmic time *t* is shown in Fig. 2. Models with ω crossing −1 near the past have been mildly favored by the analysis on the nature of dark energy from recent observations (for example see Astier et al.

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[56]). SNeIa alone favors a ω larger than −1 in the recent past and less than −1 today, regardless of whether using the thesis of a flat universe (Astier et al. [56]; Nojiri and Odintsov [57]) or not (Dicus and Repko [58]).It is observed that the EoS parameter is an increasing function of time, and the rapidity of its growth at the early stage of the Universe. Later on it tends zero. The observational limits on the equation of state parameter ω form SNIa data are $-1.67<\omega<-0.62$ (Knop et al [59]) and that form a combination of SNIa data with CMB anisotropy and galaxy clustering statistics are $-1.3<\omega<-0.79$ (Tegmark et al.[60]). Inflation at an early scales the parameter ω, which combined with the above data and dark energy constraint $\omega>-1$ suggests a physical condition.

The expression for the matter –energy density Ω_m and dark-energy density Ω_D are given by

$$\Omega_m=\frac{\rho_m}{3H^2}=\frac{\rho_0 t^2 (n_0 t^n)^{-3(1+\omega_m)}}{3n^2} \tag{30}$$

and

$$\Omega_D=\frac{\rho_D}{3H^2}=\frac{\left\{\begin{array}{l}-\frac{3(n^2-n)}{t^2}\left[bm\left(\frac{12n^2-6n}{t^2}\right)^{m-1}\right]+\frac{1}{2}\left[\left(\frac{12n^2-6n}{t^2}\right)+b\left(\frac{12n^2-6n}{t^2}\right)^{m}\right]\\ +3\frac{n}{t}\left[bm(m-1)\left(\frac{12n^2-6n}{t^2}\right)^{m-2}\left(\frac{12n-24n^2}{t^3}\right)\right]\end{array}\right\}}{\frac{3n^2}{t^2}}-\frac{\rho_0 t^2 (n_0 t^n)^{-3(1+\omega_m)}}{3n^2} \tag{31}$$

Equations (30) and (31) reduce to

$$\Omega=\Omega_m+\Omega_D=\left\{\begin{array}{l}-\frac{3(n^2-n)}{3n^2}\left[bm\left(\frac{12n^2-6n}{t^2}\right)^{m-1}\right]+\frac{t^2}{6n^2}\left[\left(\frac{12n^2-6n}{t^2}\right)+b\left(\frac{12n^2-6n}{t^2}\right)^{m}\right]\\ +\frac{t}{n}\left[bm(m-1)\left(\frac{12n^2-6n}{t^2}\right)^{m-2}\left(\frac{12n-24n^2}{t^3}\right)\right]\end{array}\right\} \tag{32}$$

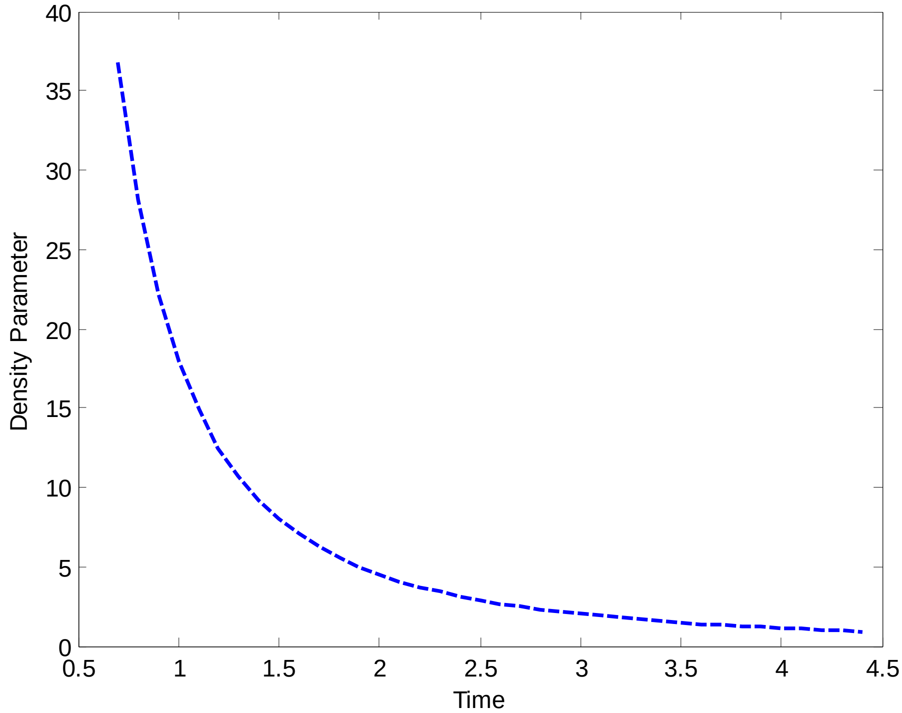

Figure 3 The plot of Density parameter vs *t*.

From the right hand side of Eq. (32), it is clear that in flat universe, at late time we see $\Omega \rightarrow 1$. This result is compatible with the observational results. Since our model predicts a flat universe for large times and the present-day universe is very close to flat, the derived model is also compatible with the observational results. The variation of density parameter with cosmic time has been shown in Fig. 3.

The deceleration parameter is given by

$$q = \frac{1-n}{n}, \tag{33}$$

which is constant .

The deceleration parameter shows signature flipping on the constraints on n. It may be noted that though the current observations of SNe Ia and CMB favor accelerating models $q < 0$, it cannot be altogether ruled out the decelerating ones which are also consistent with these observations (Vishwakarma [61]).

The expansion of the Universe is accelerating at present and it was decelerating in the past with a transition redshift of about 0.5 was disclosed by the recent observations of SN Ia (Torny et. al.[62]) and CMB anisotropies (Bennett et. al.[63]. It is therefore expected a signature flipping in the Deceleration Parameter for the Universe which was decelerating in the past and is accelerating at the present time (T. Padmanabhan and T. Roychowdhury[64]). The values of the deceleration parameter separate decelerating ($q > 0$) from accelerating ($q < 0$) periods in the evolution of the universe. The present values of the deceleration parameter obtained from observations are $-1.27 \leq q \leq 2$ (Schuecker et al. [65]). Studies of galaxy counts from redshift surveys provide a value of $q = 0.1$, with an upper limit of $q < 0.75$ (Schuecker et al. [65]). Recent observations show that the deceleration parameter of the universe is in the range $-1 \leq q \leq 0$ i.e. $q \approx -0.77$ (Cunha et al. [66]).

5. Interacting two fluid models

Secondly the interaction between dark and barotropic fluids has been considered. For this purpose we can write the continuity equations for dark fluid and barotropic fluid as

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}\left(\rho_m + p_m\right) = Q, \tag{34}$$

and

$$\dot{\rho}_D + 3\frac{\dot{a}}{a}\left(\rho_D + p_D\right) = -Q. \tag{35}$$

The quantity Q expresses the interaction between the dark components. The sign of Q determines the direction of energy transfer. A positive Q corresponds to the transfer of energy from dark energy to substance and also the different manner around for a negative one. Since we are interested in an energy transfer from the dark energy to dark matter, we consider $Q > 0$. $Q > 0$ ensures that the second law of thermodynamics stands fulfilled Pavon and Wang [67].Observationally the Abell cluster A586 provides evidence of the interaction between dark matter and dark energy (Bertolami et al. [68]). In the context of theory and physics, it's customary and appealing to interpret the dark energy as some type of particles that move with the particles of the quality model terribly debile. The weakness of the interaction is needed since dark energy particles haven't been made within the accelerators and since dark energy has not however been decayed into lighter or massless fields like photons.

Recently, Cai and Su [69] investigated the interaction in a way independent of specific interacting forms by use of observational data (SN Ie,BAO, CMB and Hubble parameter). They found that the sign of interaction Q changed in the approximate redshift range of $0.45 \leq z < 0.9$.

Here we emphasize that the continuity equations (34) and (35) imply that the interaction term (Q) should be proportional to a quantity with units of inverse of time .i.e. $Q\alpha\frac{1}{t}$. Therefore, a first and natural candidate can be the Hubble factor H multiplied with the energy density. Following Amendola et al [70] and Gou et al [71], we consider

$$Q = 3H\sigma\rho_m, \tag{36}$$

where σ is a coupling constant.

From the observational data of Gold SNeIa samples, CMB data from the WMAP satellite and the Baryonic Acoustic Oscillations (BAO) from the Sloan Digital Sky Survey (SDSS), it is calculable that the coupling parameter between matter and Dark Energy should be a little positive value of the order of unity, that satisfies the need for finding the cosmic coincidence drawback and also the constraints given by the second law of thermodynamics (Feng et al. [72]).In principle σ may be positive or negative. According to Wei and Cai [73] the cases with positive σ have physically richer phenomena. If σ = 0 then a non-interacting situation is obtained.

Using equation (36) in equation (34) and after integrating the resulting equation , we obtain

$$\rho_m = \rho_0 a^{-3(1+\omega_m-\sigma)}, \tag{37}$$

where ρ_o is an integrating constant.

By using equation (37) in equations (6) and (7), we first obtain the ρ_D and p_D in term of scale factor $a(t)$

$$\rho_D = -3\frac{\ddot{a}}{a}F + \frac{1}{2}f(R) + 3\frac{\dot{a}}{a}\dot{F} - \rho_0 a^{-3(1+\omega_m-\sigma)} \tag{38}$$

$$p_D = \left[2\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a}\right]F - \frac{1}{2}f(R) - 2\frac{\dot{a}}{a}\dot{F} - \ddot{F} - \rho_0(\omega_m - \sigma)a^{-3(1+\omega_m-\sigma)}. \tag{39}$$

5.1. Model

The super gravity inspired model (Noakes [53],M.Sharif and S.Arif [54]) given by equation (23) has been discussed.

Using equations (22),(24),(25),(26) in equations (38) and (39) , we have

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$$\rho_D=\left\{\begin{array}{l}-\dfrac{3(n^2-n)}{t^2}\left[bm\left(\dfrac{12n^2-6n}{t^2}\right)^{m-1}\right]+\dfrac{1}{2}\left[\left(\dfrac{12n^2-6n}{t^2}\right)+b\left(\dfrac{12n^2-6n}{t^2}\right)^{m}\right]\\ +3\dfrac{n}{t}\left[bm(m-1)\left(\dfrac{12n^2-6n}{t^2}\right)^{m-2}\left(\dfrac{12n-24n^2}{t^3}\right)\right]-\rho_0(n_0t^n)^{-3(1+\omega_m-\sigma)}\end{array}\right\},\quad(40)$$

In both non-interacting and interacting cases, it is observed that energy density is a decreasing function of time which approaches a small positive value at late times and never goes to infinity. Thus, in both cases the Universe is free from the Big Rip.

$$p_D=-\left\{\begin{array}{l}-\left[\dfrac{3n^2-n}{t^2}\right]\left[bm\left(\dfrac{12n^2-6n}{t^2}\right)^{m-1}\right]+\dfrac{1}{2}\left[\left(\dfrac{12n^2-6n}{t^2}\right)+b\left(\dfrac{12n^2-6n}{t^2}\right)^{m}\right]\\ +2\dfrac{n}{t}\left[bm(m-1)\left(\dfrac{12n^2-6n}{t^2}\right)^{m-2}\left(\dfrac{12n-24n^2}{t^3}\right)\right]+\\ bm(m-1)\left[(m-2)\left(\dfrac{12n^2-6n}{t^2}\right)^{m-3}\left(\dfrac{12n-24n^2}{t^3}\right)^2-\left(\dfrac{12n^2-6n}{t^2}\right)^{m-2}\left(\dfrac{72n^2-36n}{t^4}\right)\right]\\ +\rho_0(\omega_m-\sigma)(n_0t^n)^{-3(1+\omega_m-\sigma)}\end{array}\right\}\quad(41)$$

By using equations (40) and (41) in equation (10), we find the equation of state of dark field in term of time as

$$\omega_D=-\frac{\left\{\begin{array}{l}-\left[\dfrac{3n^2-n}{t^2}\right]\left[bm\left(\dfrac{12n^2-6n}{t^2}\right)^{m-1}\right]+\dfrac{1}{2}\left[\left(\dfrac{12n^2-6n}{t^2}\right)+b\left(\dfrac{12n^2-6n}{t^2}\right)^{m}\right]\\ +2\dfrac{n}{t}\left[bm(m-1)\left(\dfrac{12n^2-6n}{t^2}\right)^{m-2}\left(\dfrac{12n-24n^2}{t^3}\right)\right]+\\ bm(m-1)\left[(m-2)\left(\dfrac{12n^2-6n}{t^2}\right)^{m-3}\left(\dfrac{12n-24n^2}{t^3}\right)^2-\left(\dfrac{12n^2-6n}{t^2}\right)^{m-2}\left(\dfrac{72n^2-36n}{t^4}\right)\right]\\ +\rho_0(\omega_m-\sigma)(n_0t^n)^{-3(1+\omega_m-\sigma)}\end{array}\right\}}{\left\{\begin{array}{l}-\dfrac{3(n^2-n)}{t^2}\left[bm\left(\dfrac{12n^2-6n}{t^2}\right)^{m-1}\right]+\dfrac{1}{2}\left[\left(\dfrac{12n^2-6n}{t^2}\right)+b\left(\dfrac{12n^2-6n}{t^2}\right)^{m}\right]\\ +3\dfrac{n}{t}\left[bm(m-1)\left(\dfrac{12n^2-6n}{t^2}\right)^{m-2}\left(\dfrac{12n-24n^2}{t^3}\right)\right]-\rho_0(n_0t^n)^{-3(1+\omega_m-\sigma)}\end{array}\right\}}$$

(42)

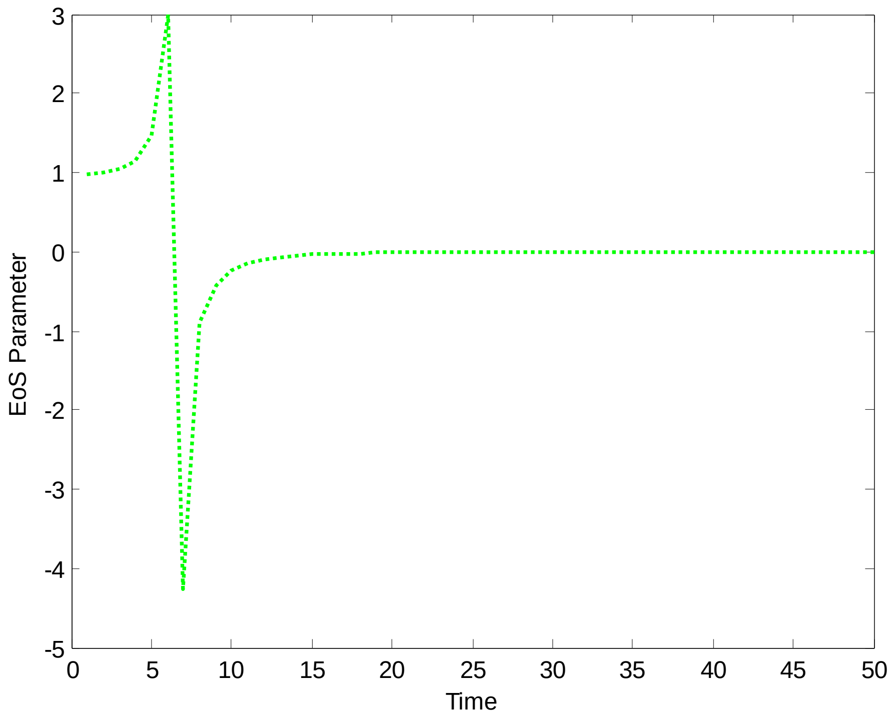

Figure 4 The plot of EoS parameter vs t.

It is observed that the EoS parameter of flat universe is varying in quintessence $(\omega_D > -0.5)$, phantom $(-1 < \omega_D < -0.5)$ and super phantom $(\omega_D > -0.3)$ regions respectively.

The expression for the matter –energy density Ω_m and dark-energy density Ω_D are given by

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{\rho_0 t^2 (n_0 t^n)^{-3(1+\omega_m-\sigma)}}{3n^2} \tag{43}$$

and

$$\Omega_D = \frac{\rho_D}{3H^2} = \frac{\left\{ \begin{array}{l} -\frac{3(n^2-n)}{t^2}\left[bm\left(\frac{12n^2-6n}{t^2}\right)^{m-1} \right] + \frac{1}{2}\left[\left(\frac{12n^2-6n}{t^2}\right) + b\left(\frac{12n^2-6n}{t^2}\right)^{m} \right] \\ +3\frac{n}{t}\left[bm(m-1)\left(\frac{12n^2-6n}{t^2}\right)^{m-2}\left(\frac{12n-24n^2}{t^3}\right) \right] \end{array} \right\}}{\frac{3n^2}{t^2}} - \frac{\rho_0 t^2 (n_0 t^n)^{-3(1+\omega_m-\sigma)}}{3n^2}$$

(44)

Equations (43)(52) and (44)(53) reduce to

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$$\Omega=\Omega_m+\Omega_D=\left\{\begin{array}{l}-\frac{3(n^2-n)}{3n^2}\left[bm\left(\frac{12n^2-6n}{t^2}\right)^{m-1}\right]+\frac{t^2}{6n^2}\left[\left(\frac{12n^2-6n}{t^2}\right)+b\left(\frac{12n^2-6n}{t^2}\right)^{m}\right]\\ +\frac{t}{n}\left[bm(m-1)\left(\frac{12n^2-6n}{t^2}\right)^{m-2}\left(\frac{12n-24n^2}{t^3}\right)\right]\end{array}\right\} \quad (45)$$

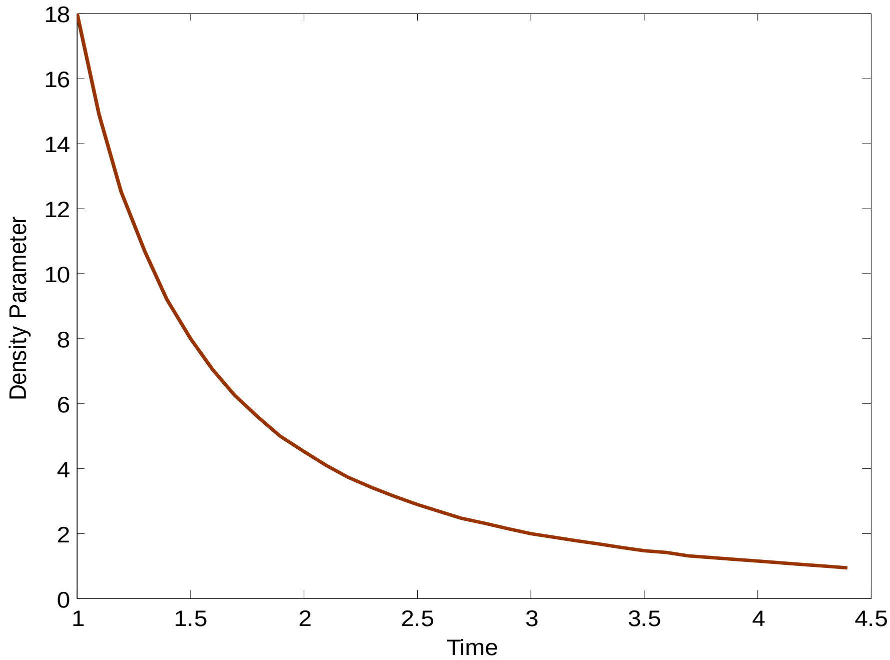

Figure 5 The plot of Density Parameter vs t.

Therefore, it is observe that in interacting case the density parameter has the same properties as in non-interacting case.

The mean Hubble parameter is given by

$$H=\frac{n}{t}. \quad (46)$$

The spatial volume of the universe is given by

$$V=n_0^3 t^{3n} \quad . \quad (47)$$

The expansion scalar is given by

$$\theta=\frac{3n}{t}. \quad (48)$$

It can be observed that the spatial volume increases as t increases while the scalar of expansion and Hubble parameter decreases. At the initial epoch, the physical quantities θ , H diverge. Hence model start expanding with a Big Bang at $t=0$. It is interesting to note that our investigations resemble with the physical behavior of

parameters of Sharif and Kausar [74].It is observed that H is a function of t and we conclude that model is not steady-state.
The mean anisotropic parameter is

$$A_m = 0 . \tag{49}$$

The shear scalar is given by

$$\sigma^2 = 0 . \tag{50}$$

It is observed from Eqs. (49) and (50) that the model in this theory becomes shear free during the evolution of the universe. That is, there is a transition from decelerated phase to accelerated phase in accordance with the observations (Caldwell et al. [75]). It is also interesting to note that the average anisotropy parameter vanishes so that the model does not remain anisotropic through out the evolution of the universe and the model becomes shear free which resembles with the investigations of Naidu et al. [76].

6. Conclusion:

i) The study of $f(R)$ models has been carried out by many Relativists. Also, different $f(R)$ models have been introduced to evaluate the energy density in $f(R)$ theory of gravity.

ii) In this paper, the evolution of the dark energy parameter has been studied within the scope of a spatially flat and isotropic Friedmann –Robertson-Walker (FRW) model filled with barotropic fluid and dark energy in the framework of $f(R)$ gravity.

iii) The scale factor and volume of the universe are zero at initial epoch and are increasing with passage of time. Thus the model represents an accelerated expansion of the universe with $V \to \infty$ as $t \to \infty$ and supports the observations of the Type Ia supernova (Permutter et. al.[52],Knop et. al.[59],Riess et. al.[49] ,Torny et. al.[62]) and WMAP data (Spergel et. al.[77-78]).

iv) From equations (46) and (48) ,we get Hubble parameter and expansion scalar , which indicate that the expansion rate is more (rapid) at initial times of the big bang but it slows down with passage of time and tends to zero as $t \to \infty$.

v) It is observed that in interacting and non-interacting cases both the flat universe can cross phantom region.

vi) In both non-interacting and interacting two-fluid scenario, the total density parameter (Ω) approaches to 1 for sufficiently large time which is reproducible with current observations. vii)The derived DE model represents an acceleration universe which is in good agreement with recent observations (Riess et al [49]). Our proposed solutions are physically stable and acceptable.

viii) Thus, the solutions obtained in this paper may be useful for better understanding of the characteristic of DE in the evolution of universe within the framework of FRW.

References:

1. Nojiri, S., Odintsov, S.D.: Phys. Rev. D **77**, 026007 (2008)
2. Weyl, H.: Ann. Phys. 59, 101, (1919).
3. Eddington, A.S.: Cambridge University Press, Cambridge(1923).
4. Jakubiec, A. and Kijowski, J.: Phys. Rev.D37, 1406, (1988).
5. Multam¨aki, T. and Vilja, I.: Phys. Rev. D74, 064022, (2006).
6. Capozziello, S., Nojiri, S., Odintsov, S.D., Troisi, A.: Phys. Lett. B **639**, 135 (2006)
7. Nojiri, S., Odintsov, S.D.: Phys. Rev. D **68**, 123512 (2003)
8. Nojiri, S., Odintsov, S.D.: Phys. Rev. D **74**, 086005 (2006)
9. Chiba, T.: Phys. Lett. B **575**, 1 (2003)
10. Dominguez, A.E., Barraco, D.E.: Phys. Rev. D **70**, 043505 (2004)
11. Nojiri, S., Odintsov, S.D.: Problems of Modern Theoretical Physics, pp. 266–285. TSPU Publishing,Tomsk (2008). A Volume in honour of Prof. Buchbinder, I.L. in the occasion of his 60th birthday.arXiv:0807.0685
12. Nojiri, S., Odintsov, S.D.: Phys. Rev. D **68**, 123512 (2003)
13. Nojiri, S., Odintsov, S.D.: Int. J. Geom. Methods Mod. Phys. **4**, 115(2007)
14. Faraoni, V.: Phys. Rev. D **74**, 023529 (2006)
15. Chiba, T., Smith, T.L., Erickcek, A.L.: Phys. Rev. D **75**, 124014 (2007)
16. Azadi, A., Momeni, D., Nouri-Zonoz, M.: Phys. Lett. B **670**, 210 (2008)
17. Momeni, D., Gholizade, H.: Int. J. Mod. Phys. D **18**, 1719 (2009)
18. Sharif, M., Shamir, M.F.: Mod. Phys. Lett. A **25**, 1281 (2010)
19. Sharif, M., Shamir, M.F.: Class. Quantum Gravity **26**, 235020 (2009)
20. Sharif, M., Shamir, M.F.: Gen. Relativ. Gravit. **42**, 2643 (2010)
21. Shamir, M.F.: Astrophys. Space Sci. **330**, 183 (2010)
22. Shamir, M.F.: Int. J. Theor. Phys. **503**, 637 (2011)
23. Capozziello, S., De Laurentis, M.: Phys. Rep. **509**, 167 (2011).

24. Shojai, A., Shojai, F.: Gen. Relativ. Gravit. **44**, 211 (2012).
25. Amir, M. and S.Sattar: arXiv:1312.1682v1 [gr-qc] (2013).
26. Sharif, M and Z.Zahra: Astrophys Space Sci ,(2013) DOI 10.1007/s10509-013-1545-8
27. Amir, M. and S.Naheed: arXiv:1312.6684v1 [gr-qc] (2013).
28. Kausar, H. and I. Noureen: arXiv:1401.8085v1 [gr-qc] (2014).
29. Katore,S and Shaikh,A.Y: The African Review of Physics (2014) 9:0054.
30. Xin, Z., Commun. Theor. Phys. **44**, 762 (2005).
31. Setare, M. R. , Physics Letters B, 644, 99,(2007a)
32. Setare, M. R. , European Physical Journal C, 50, 991,(2007b)
33. Setare, M. , Physics Letters B, 654, 1,(2007c)
34. Setare, M. R., & Saridakis, E. N. International Journal of Modern Physics D, 61, 087702,(2009)
35. Liang, N. M., C. J. Gao, S. N. Zhang, Chin. Phys. Lett. **26**, 069501 (2009)
36. Setare, M. R., J. Sadeghi, A. R. Amani, Phys. Lett. B. **673** 241 (2009)
37. Sheykhi, A., & Setare, M. R. International Journal of Theoretical Physics, 49, 2777,(2010)
38. Hassan, A., Anirudh, P., & Bijan, S., Chinese Physics Letters, 28, 039801,(2011)
39. Amirhashchi, H., Pradhan, A., & Saha, B. , Chinese Physics Letters, 28, 039801(2011a)
40. Amirhashchi, H., Pradhan, A., & Zainuddin, H. International Journal of Theoretical Physics, 50, 3529,(2011b)
41. Pradhan, A., Amirhashchi, H., & Saha, B., Ap&SS, 333, 343,(2011)
42. Saha, B., Amirhashchi, H., & Pradhan, A., Ap&SS, 342, 257,(2012)
43. Singh, T., & Chaubey, R. RAA(Research in Astronomy and Astrophysics), 12, 473,(2012)
44. Granda, L.N., Oliveros, A.: Phys. Lett. B **669**, 275 (2008)
45. Darabi, F.: Astrophys Space Sci ,343:499–504, (2013)
46. Saadat, H., Pourhassan, B.: Astrophys Space Sci , 343:783–786,(2013)
47. Xu, Y.D. , Huang, Z.G.: Astrophys Space Sci , 343:807–811,(2013)
48. Singh, J.P. , Singh, Pratibha, Bali Raj: Int J Theor Phys, 51:3828–3838,(2012)
49. Riess, A. G. et al. (Supernova Search Team Collaboration),Astron. J., **116**, 1009, 1998.
50. Riess, A. G. et al. (Supernova Search Team Collaboration), Astrophys. J., **607**, 665, 2004,.

51. Riess, A. G. et al., Astrophys. J., **659**, 98, 2007.
52. Perlmutter, S. et al. (Supernova Cosmology Project Collaboration), Astrophys. J., **517**,565,1999.
53. Noakes, D.R.: J. Math. Phys. **24**, 1840 (1983)
54. M.Sharif and S.Arif : Astrophys Space Sci , 342:237–243 (2012).
55. Sotiriou and Faraoni:Rev. Mod. Phys. 82, 451 (2010),[arXiv:0805.1726 [gr-qc]].
56. Astier, P. et al. (The SNLS Collaboration),Astron. Astrophys., **447**, 31,2006.
57. Nojiri, S., Odintsov, S.D.: Gen. Relativ. Gravit. **38**, 1285 (2006).
58. Dicus, D.A., Repko, W.W.: Phys. Rev. D **70**, 083527 (2004).
59. Knop, R.A., et al.: Astrophys. J. 598, 102 (2003)
60. Tegmark, M., et al.: Astrophys. J. **606**, 702 (2004)
61. Vishwakarma, R. G. 2003, MNRAS, 345, 545 (astro-ph/0302357)
62. Tonry, J.L., et al.: Astrophys. J. 594, 1 (2003)
63. Bennett, C.L., et al.: Astrophys. J. Suppl. Ser. **148**, 1 (2003)
64. Padmanabhan, T., Roychowdhury, T.: Mon. Not. R. Astron. Soc. 344, 823 (2003)
65. Schuecker, R., et al.: Astrophys. J. **496**, 635 (1998)
66. Cunha, C.E., Lima, M., Ogaizu, H., Frieman, J., Lin, H.: Mon. Not. R. Astron. Soc. **396**, 2379 (2009)
67. Pavon, D., Wang, B.: Gen. Relativ. Gravit. **41**, 1 (2009)
68. Bertolami, O., Gil Pedro, F., Le Delliou, M.: Phys. Lett. B **654**, 165 (2007)
69. Cai, R.G., Su, Q.P.: Phys. Rev. D **81**, 103514 (2010)
70. Amendola L, L., Camargo Campos, G., Rosenfeld, R.: Phys. Rev. D **75**, 083506 (2007)
71. Guo, Z.K., Ohta, N., Tsujikawa, S.: Phys. Rev. D **76**, 023508 (2007)
72. Feng, C., et al.: Phys. Lett. B **665**, 111 (2008)
73. Wei, H., Cai, R.-G.: Eur. Phys. J. C **59**, 99 (2009)
74. Sharif, M and Rizwana Kausar, H.: arXiv:1101.3372v1 [gr-qc] 18 Jan 2011.
75. Caldwell, R.R., Komp, W., Parker, L., Vanzella, D.A.T.: Phys. Rev. D **37**, 023513 (2006)
76. Naidu,R.L.,Reddy, D.R.K.,Ramprasad,T.,Ramana, K.V.: Astrophys Space Sci;DOI 10.1007/s10509-013-1540-0
77. Spergel, D. N. et al. (WMAP Collaboration), 2003, Astrophys. J. Suppl., **148**, 175.
78. Spergel, D. N. et al. (WMAP Collaboration), 2007, Astrophys. J. Suppl., **170**, 377.
79. Perlmutter et al., Astrophys. J. 483, 565 (1997).