International Journal of Universal Mathematics and Mathematical Sciences
ISSN: 2454-7271 Volume: 02, Issue: 01, Pages:28-39,
Published:June,2016 Web: www.universalprint.org ,
Title Key: Radial Symmetry of Solutions of System of Nonlinear..…

Radial Symmetry of Solutions of System of Nonlinear Elliptic Boundary Value Problems

D. P. Patil
Department of Mathematics,
Arts, Science and Commerce College
Saikheda Tal Niphad. Dist: Nasik,
Email: sdinkarpatil95@gmail.com

Abstract

Radial symmetry of positive solutions of system of nonlinear elliptic bound- ary value problems in R^n is studied. We apply the moving plane method based on maximum principle to obtain our result of symmetry of solutions on unbounded domain R^n.

Keywords: Maximum principles, Moving plane method, Radial symmetry; System of nonlinear elliptic boundary value problems.

Subject Classification: 35B06, 35B09, 35J25, 35B50.

1. Introduction:

$$\Delta u + f(|x|; u) = 0 \text{ in } R^n$$
and
$$u(x) \to 0 \text{ as } |x| \to \infty.$$

Furthermore, Naito [18] also studied the problem of radial symmetry of classical solutions of semilinear elliptic equations $\Delta u + V(|x|)\, e^u = 0$ in R^2, by the moving plane method. Horvata, et al. [13] studied the existence of positive spherically symmetric solutions of polyharmonic boundary value problems. They study the system by relating it to the corresponding system of singular integro differential equations of the first order. Ma and Liu [15] investigated the symmetry properties of positive solutions of semilinear elliptic system . Pucci, Sciunzi and Serrin [20] studied symmetry of solutions of degenerate quasilinear elliptic problems by applying comparison principle. L. Montoro, B. Sciunzi [16] proved radial symmetry in balls for regular solutions of a class of quasilinear elliptic system in

International Journal of Universal Mathematics and Mathematical Sciences
ISSN: 2454-7271 Volume: 02, Issue: 01, Pages:28-39,
Published:June,2016 Web: www.universalprint.org ,
Title Key: Radial Symmetry of Solutions of System of Nonlinear..…

nonvariational form. Covei [2] proved necessary and sufficient conditions for a positive radial solution of semilinear ellipic problem. In 2012, Damascelli et al. [3] proved symmetry of solutions of semilinear cooperative elliptic system in unit ball and in annulus in R^n. Abduragimov [1] proved the existence and uniqueness of positive radially symmetric solutions of the Dirichlet problem for a nonlinear elliptic system. Farina [9] proved the symmetry result for the system,

$$\Delta u = uv^2; \Delta v = vu^2; u \geq 0; v \geq 0 \text{ in } R^n:$$

Damascelli and Pacella [4] proved solutions having Morse index $j \leq N$ are foliated schwartz symmetric by using symmetrization for a semilinear elliptic system. Recently Dhaigude and Patil [5] studied the radial symmetry of positive solutions of semilinear elliptic problem in unit ball. Also, Dhaigude and Patil [6], [7], [8] obtained symmetric results for semilinear elliptic boundary value problems and system of nonlinear boundary value problems, using moving plane method.

In this paper we study the radial symmetry of positive solutions for system of nonlinear elliptic boundary value problem in R^n. We consider the nonlinear elliptic boundary value problem of the form

$$\begin{aligned} \Delta u + f(|x|; u; v) = 0 \\ \Delta v + g(|x|; u; v) = 0 \end{aligned} \quad \text{in } R^n \tag{1.1}$$

and

$$u(x) \to 0, \; v(x) \to 0 \text{ as } |x| \to \infty. \tag{1.2}$$

These type of systems occur in many models of physics, where study of symmetry property is important.Our approach is based on the maximum principle in unbounded domains together with the moving plane method. This approach helps us to prove our results..We organise the paper as follows: In section 2, the preliminary results and some useful lemmas are proved. The symmetric result is proved and some illustrative examples are given in the last section.

2. Preliminaries

In this section, _rst we state some basic lemmas and boundary maximum principle which are useful to prove our main result.

Lemma 2.1 *[12]Hopf boundary lemma : Suppose that Ω satisfies the interior sphere condition at $x_0 \in \partial\Omega$. Let L be uniformly elliptic with $c(x) \leq 0$ where*

International Journal of Universal Mathematics and Mathematical Sciences
ISSN: 2454-7271 Volume: 02, Issue: 01, Pages:28-39,
Published:June,2016 Web: www.universalprint.org ,
Title Key: Radial Symmetry of Solutions of System of Nonlinear..…

$$L(u)=\sum_{i,j=1}^{n} a_{ij}(x)\frac{\partial^2(x)}{\partial x_i \partial x_j}+\sum_{j=1}^{n} b_i(x)\frac{\partial u}{\partial x_j}+c(x)u \quad in\ \Omega.$$

If $u\in C^2(\Omega)\bigcap C(\overline{\Omega})$ *satisfies* $L(u)\ge 0$ *and* $\max_{\overline{\Omega}} u(x)=u(x_0)$ *, hen either* $u = u(x0)$ *on* Ω *or* $\liminf_{t\to 0}\frac{u(x)-u(x_0+t\nu)}{t}>0$. *For every direction* ν *pointing into an interior sphere. If* $u\in C^1\subset\Omega\bigcap\{0\}$ *then* $\frac{\partial u}{\partial \nu}(x_0)<0$

Lemma 2.2 *[17] Let* Ω *be an unbounded domain in* R^n*, and let L denote the uniformly elliptic differential operator of the form*

$$L(u)=a^{ij}(x)\partial_{ij}u+b^i(x)\partial_i(u)+c(x)u$$

where a^{ij}, b^i, $c\in L^\infty(\Omega)$. *Suppose that* $u\neq 0$ *satisfies*
$L(u)\le 0$ *in* Ω *and* $u\ge 0$ *on* $\partial\Omega$:
Furthermore, suppose that there exist a function w such that $w>0$ *on* $\Omega\bigcup\partial\Omega$ *and* $L(w)\le 0$ *in* Ω.

If $\frac{u(x)}{w(x)}\to 0$ *as* $|x|\to\infty$, $x\in\Omega$, *then* $u>0$ *in* Ω.

Theorem 2.1 *[19] Let* $u(x)$ *satisfies differential inequality*

$$L(u)\ge 0$$

in a domain Ω *where L is uniformly elliptic. If there exist a function w(x) such that,* $w(x)>0$ *on* $\Omega\bigcup\partial\Omega$ *and satisfies the differential inequality*

$$L(w)\le 0 \text{ in } \Omega,$$

then $\frac{u(x)}{w(x)}$ *can not attain a non negative maximum at a point p on* $\partial\Omega$*, which lies on the boundary of a ball in* Ω *and if* $\frac{u(x)}{w(x)}$ *is not constant then,* $\frac{\partial}{\partial\nu}(\frac{u}{w})>0$
$\sum_{\lambda}$

$$\frac{\partial}{\partial\nu}(\frac{u}{w})>0 \qquad at\ P.$$

where $\frac{\partial}{\partial\nu}$ *is any outward directional derivative.*

3. Main Results

In this section, we prove our main result. We define the plane T_λ, for a real number λ as follows $T_\lambda = \{x : x = (x1; x2; x3; :::; xn); x1 = \lambda\}$, which is perpendicular to $X1$-axis. We will move this plane continuously normal to itself to new position till it begins to intersect Ω. After that point the plane advances in Ω along $X1$- axis and cut of cap $\sum_\lambda$; which is the portion of Ω and lies in the same side of the plane T_λ as the original plane T. Let $\sum_\lambda = \{ x : x1 < \lambda ; x \in \Omega\}$: Let $x^\lambda = (2\lambda - x1; x2; x3; :::; xn)$ be the reflection of the point $x = (x1; x2; x3; :::; xn)$, about the plane T_λ. Define $w1;\lambda(x) = u(x) - u(x_\lambda)$, and $w2; \lambda (x) = v(x) - v(x_\lambda)$. We have $|x^\lambda| \geq |x|$ and $u(x_\lambda) = u(2\lambda - x1; x2; x3; :::; xn)$. Also define set $\wedge = \{ \lambda \in (0;\infty) : w1;\lambda(x) > 0; w2;\lambda (x) > 0\}$ for $x \in \sum_\lambda$.

Now, we prove our main result,

Theorem 3.1 *Suppose that*

⟨i⟩ (u; v) $\in C^2$ is a positive solution of the system of nonlinear elliptic boundary value problem (1.1)-(1.2),

⟨ii⟩ functions f and g are continuous in all its variables and C1 in u ≥ 0, v ≥ 0,

⟨iii⟩ f(|x|, u(x); v(x1; x2; x3; :::; xi-1; 2λ - xi; xi+1; :::xn))= f(|x|, u(x); v(x1; x2; x3; :::; xi-1; xi; xi+1; :::xn)) , for all 1 ≤ i ≤ n,

⟨iv⟩ g (|x| ; u(x1; x2; x3; :::; xi□1; 2λ - xi; xi+1; :::; xn); v(x)) = g (|x| ; u(x1; x2; x3; :::; xi□1; xi; xi+1; :::; xn); v(x)) for all 1 ≤ i ≤ n,

⟨v⟩ functions f and g are nonincreasing in |x|, for each fixed u ≥ 0, v ≥ 0.

Further we define U ,V and Φ such as

$$U(r) = \sup \{u(x) : |x| \geq r\} \tag{3.1}$$

$$V (r) = \sup\{ v(x) : |x| \geq r\} \tag{3.2}$$

$$\Phi (|x|) = \sup \{ fu(|x\}; u; v); gv(|x|; u; v) : 0 \leq u(x) \leq U(r); 0 \leq v(x) \leq V (r)\} \tag{3.3}$$

Furthermore assume that there exist positive functions z1; z2 on |x| ≥ _ R0; for some positive constant R0 satisfying differential inequalities

$$\begin{aligned} &\Delta z_1 + \Phi(|x|)z_1 \leq 0 \\ &\qquad\qquad\qquad In\ |x| > R_0 \\ &\Delta z_2 + \Phi(|x|)z_2 \leq 0 \end{aligned} \tag{3.4}$$

and

$$\lim_{|x|\to\infty} \frac{U(x)}{z_1(x)} = 0 \tag{3.5}$$

$$\lim_{|x|\to\infty} \frac{V(x)}{z_2(x)} = 0 \tag{3.6}$$

Then (u; v) is radially symmetric about some x0 in Rn and ur < 0, ur < 0 for r = |x - x0| > 0.

To prove our result following lemmas are useful. First we prove them.

Lemma 3.1 Let $\lambda \geq 0$, then w1;λ(x) and w2;λ(x) satisfies differential inequalities

$$\Delta w_{1,\lambda}(x) + C_{1,\lambda}(x) w_{1,\lambda}(x) \leq 0 \quad \text{in } \sum{}_{\lambda}$$

$$\Delta w_{2,\lambda}(x) + C_{2,\lambda}(x) w_{2,\lambda}(x) \leq 0 \quad \text{in } \sum{}_{\lambda}$$

Where

$$C_{1,\lambda}(x) = \int_0^1 f_u(|x|, \mathrm{u}(\mathrm{x}^{\lambda}) + \mathrm{t}[\mathrm{u}(\mathrm{x}) - \mathrm{u}(\mathrm{x}^{\lambda})];\ \mathrm{v}(\mathrm{x}))dt$$

$$C_{2,\lambda}(x) = \int_0^1 g_v(|x|, u(\mathrm{x}),\ \mathrm{v}(\mathrm{x}^{\lambda}) + \mathrm{t}[\mathrm{v}(\mathrm{x}) - \mathrm{v}(\mathrm{x}^{\lambda})])dt$$

Proof: Since f(|x|; u; v) and g(|x|; u; v) are nonincreasing in |x| and $|x^{\lambda}| > |x|$ for $x \in \sum_{\lambda}$ hold. We observe that $u(x^{\lambda})$ and $v(x^{\lambda})$ satisfy the equations

$$\Delta u(x^{\lambda}) + f(|x^{\lambda}|, u(x^{\lambda}), v(x^{\lambda}) = 0 \quad \text{in } \sum{}_{\lambda} \tag{3.7}$$

$$\Delta v(x^{\lambda}) + g(|x^{\lambda}|, u(x^{\lambda}), v(x^{\lambda}) = 0 \quad \text{in } \sum{}_{\lambda} \tag{3.8}$$

Subtracting (3.7) from first equation of (1.1) we get

$$\Delta w_{1,\lambda}(x) + C_{1,\lambda}(x) w_{1,\lambda}(x) \leq 0 \quad \text{in } \sum{}_{\lambda}$$

where

$$C_{1,\lambda}(x) = \int_0^1 f_u(|x|, \mathrm{u}(\mathrm{x}^{\lambda}) + \mathrm{t}[\mathrm{u}(\mathrm{x}) - \mathrm{u}(\mathrm{x}^{\lambda})];\ \mathrm{v}(\mathrm{x}))dt$$

Similarly subtracting (3.8) from second equation of (1.1) we can prove that,

$$\Delta w_{2,\lambda}(x) + C_{2,\lambda}(x) w_{2,\lambda}(x) \leq 0 \quad \text{in } \sum{}_{\lambda}$$

where

$$C_{2,\lambda}(x) = \int_0^1 g_v(|x|, u(x), v(x^\lambda) + t[v(x) - v(x^\lambda)])dt$$

The proof of the lemma is completed.

Lemma 3.2 *Let* $\lambda > 0$, If $w_{1,\lambda}(x) > 0$, and $w_{2,\lambda}(x) > 0$ in $\sum_\lambda \cap \overline{B_0}$, Then $\lambda \in \Lambda$.

Proof: By using assumptions and lemma 3.1, we have

$$\Delta w_{1,\lambda}(x) + C_{1,\lambda}(x) w_{1,\lambda}(x) \le 0$$

$$\text{in } \sum_\lambda \setminus \overline{B_0}$$

$$\Delta w_{2,\lambda}(x) + C_{2,\lambda}(x) w_{2,\lambda}(x) \le 0$$

and

$w_{1,\lambda}(x) > 0$, and $w_{2,\lambda}(x) > 0$ on $\partial(\sum_\lambda \setminus \overline{B_0})$.

Since $U(r)$ and $V(r)$ are nonincreasing, we have

$$0 \le u(x^\lambda) + t(u(x) - u(x^\lambda) \le U(|x|)$$

$$0 \le t \le 1$$

$$0 \le v(x^\lambda) + t(v(x) - v(x^\lambda) \le V(|x|)$$

Then we observe that

$$C_{1,\lambda}(x) \le \int_0^1 \Phi(|x|)dt \le \Phi(|x|) \quad \textbf{in } \sum_\lambda$$

Similarly we can show that,

$$C_{2,\lambda}(x) \le \int_0^1 \Phi(|x|)dt \le \Phi(|x|) \quad \textbf{in } \sum_\lambda$$

From (3.4) we have

$$\Delta z_1 + C_{1,\lambda}(x) \le 0 \quad \textbf{in } \sum_\lambda \setminus \overline{B_0}$$

$$\Delta z_2 + C_{2,\lambda}(x) \le 0 \quad \textbf{in } \sum_\lambda \setminus \overline{B_0}$$

and

$$\frac{w_{1,\lambda}(x)}{z_1(x)} \to 0 \,, \quad \frac{w_{2,\lambda}(x)}{z_2(x)} \to 0 \quad \textbf{as } |x| \to \infty, \textbf{ for } x \in \sum_\lambda \setminus \overline{B_0}$$

Hence by Lemma (2.2)

$w_{1,\lambda}(x) > 0$, and $w_{2,\lambda}(x) > 0$ in $\sum_\lambda \setminus \overline{B_0}$ **.**

From this it follows that,

$w_{1,\lambda}(x) > 0$, and $w_{2,\lambda}(x) > 0$ in $\sum_\lambda$

Therefore $\lambda \in \Lambda$ .This completes the proof

Lemma 3.3 Let $\lambda \in \Lambda$, *Then* $\frac{\partial u}{\partial x_1} < 0$ *and* $\frac{\partial v}{\partial x_1} < 0$ *on* **T**λ.

Proof: By Lemma 3.1 we have

$$\Delta w_{1,\lambda}(x) + C_{1,\lambda}(x) w_{1,\lambda}(x) \le 0 \quad \textbf{in } \sum{}_{\lambda}$$

$$\Delta w_{2,\lambda}(x) + C_{2,\lambda}(x) w_{2,\lambda}(x) \le 0 \quad \textbf{in } \sum{}_{\lambda}$$

Since $w_{1,\lambda}(x)$ = 0 on T_λ by Hopf's boundary lemma we have $\frac{\partial w_{1,\lambda}(x)}{\partial x_1} < 0$ and $\frac{\partial w_{2,\lambda}(x)}{\partial x_1} < 0$ on T_λ . We have $\frac{\partial w_{2,\lambda}(x)}{\partial x_1} = 2\frac{\partial v}{\partial x_1}$ and $\frac{\partial w_{2,\lambda}(x)}{\partial x_1} = 2\frac{\partial v}{\partial x_1}$.

Therefore $\frac{\partial u}{\partial x_1}$ **< 0** $\frac{\partial u}{\partial x_1}$ **< 0**

Now we prove theorem3.1.

Proof of the theorem:

Since $u(x)$; $v(x)$ are positive with

$$\lim_{|x|\to\infty} u(x) = 0$$

and

$$\lim_{|x|\to\infty} v(x) = 0$$

then there exist $R1;R2 > R0$ such that

$$\max\{u(x) : |x| \ge R_1\} < \min\{u(x) : |x| \le R_0\} \tag{3.9}$$

$$\max\{v(x) : |x| \ge R_2\} < \min\{v(x) : |x| \le R_0\} \tag{3.10}$$

$$w_{1,\lambda}(x) > 0, \text{ in } \overline{B_0}$$

Also from equation (3.10) we get,

$$w_{2,\lambda}(x) > 0, \text{ in } \overline{B_0}$$

Then by Lemma 3.2 we have $\lambda \in \Lambda$. Therefore $[Rm;\infty) \subset \Lambda$. Hence step-I is completed.
Step-II: In this step we prove that, if $\lambda_0 \in \Lambda$ then there exist $\in > 0$ such that $(\lambda_0 - \in; \lambda_0] \subset \Lambda$. Assume to the contrary that there exist an increasing sequence $\{\lambda_i\}$, i = 1; 2; 3; ::: such that $\lambda_i \notin \Lambda$ and $\lambda_i \to \lambda_0$ as $i \to \infty$, which contradicts to Lemma 3.2.

Therefore we have a sequence {xi} i = 1; 2; 3; ::: such that $x_i \in \sum_{\lambda i} \cap \overline{B_0}$ and $w_{1,\lambda i}(x_i) \le 0$ or $w_{2,\lambda i}(x_i) \le 0$. A subsequence $\{x_i^*\}$, converges to some point $x_0 \in \overline{\sum_{\lambda 0}} \cap \overline{B_0}$. Then $w_{1,\lambda i}(x_i) \le 0$ or $w_{2,\lambda i}(x_i) \le 0$. But in $\sum_{\lambda 0}$, we must have

$x_i \to x_i^{\lambda i}$

$w_{1,\lambda 0}(x_0) > 0$ and $y_i \to x_0$. Therefore we conclude that $x_0 \in T_{\lambda 0}$. By mean value

$u(x) \ge u(x^0)$

theorem, there exist a point yi such that

$$\frac{\partial u}{\partial x_i}(y_i) \ge 0$$

and

$$\frac{\partial v}{\partial x_i}(y_i) \ge 0$$

on the line segment joining $x_i \to x_i^{\lambda i}$ for each i = 1; 2; 3; :::. Also $y_i \to x_0$ as $i \to \infty$.

So $\frac{\partial u}{\partial x_1}(x_0) \ge 0$ and $\frac{\partial v}{\partial x_1}(x_0) \ge 0$

But by Lemma 3.1 we have

$$\frac{\partial u}{\partial x_1}(x_0) \le 0$$

and

$$\frac{\partial v}{\partial x_1}(x_0) \le 0$$

which is a contradiction and step II is completed.

Step-III: Consider the following two statements (A) and (B),

(A) $u(x) \equiv u(x^{\lambda 1})$ and $v(x) \equiv v(x^{\lambda 1})$ for some $\lambda_1 > 0$ and $\frac{\partial u}{\partial x_1} \le 0$, $\frac{\partial v}{\partial x_1} \le 0$ on T_λ for $\lambda > \lambda_1$

Or

(B) $u(x) \ge u(x^0)$ and $v(x) \ge v(x^0)$ in $\sum_{\lambda 1}$ and $\frac{\partial u}{\partial x_1} \le 0$, $\frac{\partial v}{\partial x_1} \le 0$ on T_λ for $\lambda > 0$.

We consider two cases (i)If $\lambda_1 > 0$, then we prove that statement (A) holds. (ii) If $\lambda_1 = 0$, then we prove that statement (B) holds.

Define $\lambda_1 = \min\{\lambda > 0 : [\lambda, \infty) \subset \Lambda\}$.

International Journal of Universal Mathematics and Mathematical Sciences
ISSN: 2454-7271 Volume: 02, Issue: 01, Pages:28-39,
Published:June,2016 Web: www.universalprint.org ,
Title Key: Radial Symmetry of Solutions of System of Nonlinear.….

Case (i) where $\lambda_1 > 0$.
We have

$$w_{1,\lambda 1}(x) = u(x) - u(x^{\lambda 1})$$

$$w_{2,\lambda 1}(x) = v(x) - v(x^{\lambda 1})$$

From the continuity of u and v , we have $w_{1,\lambda 1}(x) \geq 0$, $w_{2,\lambda 1}(x) \geq 0$ in $\sum_{\lambda 1}$. From lemma 3.1 it follows that

$$\left.\begin{aligned} \Delta w_{1,\lambda 1}(x) + c_{1,\lambda 1}(x) w_{1,\lambda 1}(x) \leq 0 \\ \Delta w_{2,\lambda 1}(x) + c_{2,\lambda 1}(x) w_{2,\lambda 1}(x) \leq 0 \end{aligned}\right\} \text{ in } \sum_{\lambda 1}$$

Hence by strong maximum principle we have either $w_{1,\lambda 1}(x) \geq 0$, $w_{2,\lambda 1}(x) \geq 0$ or $w_{1,\lambda 1}(x) = 0$, $w_{2,\lambda 1}(x) = 0$ in $\sum_{\lambda 1}$

Assume that $w_{1,\lambda 1}(x) > 0$, $w_{2,\lambda 1}(x) > 0$ in $\sum_{\lambda 1}$, then by lemma 3.2, $\lambda \in \Lambda$. From step-II there exists $\in > 0$ such that $(\lambda_1 - \in, \lambda_1] \subset \Lambda$.

This is contradiction to the definition of λ_1. Therefore $w_{1,\lambda 1}(x) = 0$, $w_{2,\lambda 1}(x) = 0$ in $\sum_{\lambda 1}$.

Since $(\lambda_1 , \infty) \subset \Lambda$, we have $\frac{\partial u}{\partial x_1} \leq 0$, $\frac{\partial v}{\partial x_1} \leq 0$ on T_λ for $\lambda > \lambda_1$, by Lemma 3.3, the statement (A) is follows for case (i).

Case (ii) Consider the case where $\lambda 1 = 0$. From the continuity of u and v we have $u(x) \geq u(x^0), v(x) \geq v(x^0)$ in $\sum_0$. By lemma 3.3 we get $\frac{\partial u}{\partial x_1} \leq 0$, $\frac{\partial v}{\partial x_1} \leq 0$ on T_λ for $\lambda > 0$.

Thus statement (B) holds.
If statement (B) occurs in step (III), we can repeat the previous steps I – III for negative $X1$-direction to conclude that either $(u; v)$ is symmetric in $X1$-direction about some plane $x_1 = \lambda_1 < 0$. Therefore $(u; v)$
must be symmetric in $X1$-direction about some plane and strictly decreasing away from the plane. As we may take any direction as the $X1$- direction , we conclude that $(u; v)$ is symmetric in every direction about some plane. Therefore $(u; v)$ is radially symmetric about some point $x_0 \in R^n$, $\frac{\partial u}{\partial r} < 0$, $\frac{\partial v}{\partial r} < 0$, for $r > 0$.

Example
Now, we discuss an example to illustrate theorem 3.1.

Example 3.1 *Consider the elliptic system*

$$\Delta u+(n-3)v=0 \tag{3.11}$$

$$\Delta v+3(n-5)u^2 v=0 \tag{3.12}$$

Here $f(|x|, u, v) = (n-3)v$ is linear and $f(|x|, u, v) = 3(n-5)u^2 v$ is nonlinear function.

Clearly $\Phi(|x|)=3(n-5)u^2=\dfrac{3(n-5)}{|x|^2}$ where $u=\dfrac{1}{|x|}$. Suppose $z_1(x)=\dfrac{1}{|x|^{\frac{1}{2}}}$, and $z_2(x)=\dfrac{1}{|x|^2}$. Here $z1(x)$ and $z2(x)$ satisfies the inequalities (3.4).

We also have, $\lim\limits_{|x|\to\infty}\dfrac{u(x)}{z_1(x)}=0$

Thus condition (3.5) is satisfied. similarly we can show that condition (3.6) holds for the function $z_2(x)=\dfrac{1}{|x|^2}$. Thus all the conditions of theorem 3.1 are satisfied. Therefore solutions must be radially symmetric. Clearly this system of equations have radially symmetric solutions about the origin.

4. References

[1] E. I. Abduragimov, *Uniqueness of positive radially symmetric solutions of the Dirichlet problem for nonlinear elliptic system of second order*, Mathematical notes , Jan 2013, vol.93, issue 1-2, 3-11.

[2] D. Covei, *Radial and non radial solutions for a semilinear elliptic system of Schrodinger type*, Funkcialaj Ekvacioj, 54, (2011) 439 - 449.

[3] L. Damascelli, F. Gladi, F. Pacella, *A symmetry result for semilinear cooperative elliptic systems*, Serrin, James B.(ed.) et al., Recent trends in nonlinear partial differential equations. II: Stationary problems. Workshop in honor of Patrizia Pucci's 60th birthday 'Nonlinear partial differential equations', Perugia, Italy, May 28 - June 1,2012. Providence, RI: American Mathematical Society (AMS). Contemporary Mathematics 595, (2013),187-204 .

[4] L. Damascelli, F. Pacella, *Symmetry results for cooperative elliptic systems via linearization,* SIAM J. Math. Anal. vol.45 No. 3 (2013), 1003 - 1026.

[5] D.B.Dhaigude, D.P.Patil, *Symmetry properties of solutions of nonlinear elliptic equations,* International journal of Advances in Management, Technology and engineering sciences,Vol.II ,10(11), (July 2013), 24 - 28.
[6] D.B.Dhaigude, D.P.Patil, *Symmetry of solutions of system of nonlinear elliptic boundary value problems in ball*, Journal of Global Research in Mathematical Archives, vol.2 No.4 (April 2014), 110-115.
[7] D.B.Dhaigude, D.P.Patil, *Radial symmetry of positive solutions for nonlinear elliptic boundary value problems*, Malaya J. Mat.,3(1) (2015), 23-29.
[8] D.B.Dhaigude, D.P.Patil, *On symmetric solutions of elliptic boundary value problems,* International Journal of Latest Technology in Engineering and Applied Science, vol.IV , issue 1, (Jan 2015), 48-53.
[9] A. Farina, *Some symmetry results for entire solutions of an elliptic system arising in phase separation*, Discrete Contin. Dyn. Syst. 34, No.6, (2014), 2505-2511.
[10] B.Gidas, W.M.Ni and L.Nirenberg, *Symmetry and related properties via the maximum principle,* Comm.Math.Phys.68(1979), 209-243.
[11] B.Gidas, W.M.Ni and L.Nirenberg, *Symmetry of positive solutions of nonlinear elliptic equations in Rn* , Mathematical Analysis and Applications Part A, ed.by L. Nachbin, adv. Math. Suppl. Stud. 7, Academic Press, New York, 1981, 369-402.
[12] D. Gilbarg, N.S. Trudinger; *Elliptic Partial Differential Equations of Second Order,* Springer-Verlag, Berlin, 2001.
[13] L. Horvata, J. Kraljevicb, D. Zbreve;ubrinica and V. Zbreve;upanovica, *Positive solutions of polyharmonic equations with strong dependence on the gradient complex variables and elliptic equations* , vol. 52, No. 8, Aug 2007, 693-707.
[14] H.Hopf, *Differential Geometry in the Large, Lecture notes in in Mathematics*,Vol 1000, Springer Verlag, 1983.
[15] L. Ma, B. Liu, *Symmetry results for decay solutions of elliptic systems in whole space,* Advances in Mathematics, 225 (2010) 3052 - 3063.
[16] L. Montoro, B.Sciunzi, *Symmetry results for nonvariational quasilinear elliptic systems,* Advanced Nonlinear Studies 10(2010) 939 - 955.
[17] Y. Naito, *Radial symmetry of positive solutions for semilinear elliptic equations in* R^n; J. Korean Math. Society 37 (2000) No.5, P.P. 751-761.
[18] Y. Naito, *Symmetry results for semilinear elliptic equations in* R^2; Nonlinear Analysis 47 (2001),3661-3670.
[19] M. Protter and H. Weinberger, *Maximum Principles in Differential Equations*, Springer Verlag, 1984.

[20] P.Pucci, B. Sciunzi and J. Serrin, *Partial and full symmetry of solutions of quasilinear elliptic equations, via the comparison principle,* Contemporary Mathematics, special volume dedicated to H. Brezis (2007).

[21] J.Serrin, *A symmetry problem in potential theory,* Arch.Rational Mech. Anal 43,(1971),304-318.

[22] W. Troy, *Symmetry properties in systems of semilinear elliptic equations*, J.diff.eqn.42,(1981),400-413.