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On αrω–separation axioms in topological spaces

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Abstract: The aim of this paper is to introduce and study two new classes of spaces, namely αrω–normal and αrω–regular spaces and obtained their properties by utilizing αrω–closed sets. Recall that a subset A of a topological space (X, τ) is called αrω–closed if cl(A) ⊆ U whenever A ⊆ U and U is rw– open in (X, τ) . We will present some characterizations of αrω–normal and αrω–regular spaces.

Keywords: αrω–closed set, αrω–continuous function.

1 Introduction

Maheshwari and Prasad[8] introduced the new class of spaces called s–normal spaces using semi–open sets. It was further studied by Noiri and Popa[10],Dorsett[6] and Arya[1]. Munshi[9], introduced g–regular and g– normal spaces using g–closed sets of Levine[7]. Later, Benchalli et al [3] and Shik John[12] studied the concept of g* – pre regular, g* – pre normal and ω–normal, ω–regular spaces in topological spaces. Recently, Benchalli et al [2,] introduced and studied the properties of αrω–closed sets and αrω–continuous functions.

2 Preliminaries

Throughout this paper (X, τ) , (Y, σ) (or simply X, Y) denote topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X the closure, interior and α–closure of A with respect to τ are denoted by cl(A), int(A) and αcl(A) respectively

Definition 2.1. A subset A of a topological space X is called a

(1) semi–open set [3] if A ⊆ cl(int(A)).
(2) ω–closed set[12] if cl(A) ⊆ U whenever A ⊆ U and U is semi–open in X.
(3) g–closed set[7] if cl(A) ⊆ U whenever A ⊆ U and U is open in X.

Definition 2.2. A topological space X is said to be a

(1) g–regular[10],if for each g–closed set F of X and each point $x \notin F$, there exists disjoint open sets U and V such that $F \subseteq U$ and $x \in V$.

(2) α–regular [4], if for each closed set F of X and each point $x \notin F$, there exists disjoint α– open sets U and V such that $F \subseteq V$ and $x \in U$.

(3) ω–regular[12], if for each ω–closed set F of X and each point $x \notin F$, there exists disjoint open sets U and V such that $F \subseteq U$ and $x \in V$.

Definition 2.3. A topological space X is said to be a

(1) g–normal [10], if for any pair of disjoint g–closed sets A and B, there exists disjoint open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

(2) α–normal [4], if for any pair of disjoint closed sets A and B, there exists disjoint α–open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

(3) ω–normal [12], if for any pair of disjoint ω–closed sets A and B, there exists disjoint open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Definition 2.4. [2] A topological space X is called $T_{\alpha r\omega}$-space if every αrω–closed set in it is closed set.

Definition 2.5. A function $f : X \to Y$ is called

(1) αrω–continuous [4] (resp. ω–continuous [12]) if $f^{-1}(F)$ is αrω–closed (resp. ω–closed) set in X for every closed set F of Y .

(2) αrω–irresolute [4] (resp. ω–irresolute [12]) if $f^{-1}(F)$ is αrω–closed (resp. ω–closed set in X for every αrω–closed (resp. ω– closed) set F of Y .

(3) pre–ωα–closed[4](resp. αrω–closed[ ]) if for each α–closed (resp. closed) set F of X, f(F) is an ωα–closed (resp. αrω–closed) set in Y .

Definition 2.6 A topological space X is called

i) a α–T_0 [14] if for each pair of distinct points x, y of X, there exists a α–open sets G in X containing one of them and not the other.

ii) a α–T_1 [14] if for each pair of distinct points x, y of X , there exists two α–open sets G_1, G_2 in X such that $x \in G_1$, $y \notin G_1$, and $y \in G_2$, $x \notin G_2$.

iii) a α–T_2 [14] (α– Hausdorff) if for each pair of distinct points x, y of X there exists distinct α–open sets H_1 and H_2 such that H_1 containing x but not y and H_2 containing y but not x.

3 αrω–Regular Space

In this section, we introduce a new class of spaces called αrω–regular spaces using αrω–closed sets and obtain some of their characterizations

Definition 3.1. A topological space X is said to be αrω–regular if for each αrω–closed set F and a point $x \notin F$, there exist disjoint open sets G and H such that $F \subseteq G$ and $x \in H$.

We have the following interrelationship between αrω–regularity and regularity.

Theorem 3.2. Every αrω–regular space is regular.

Proof: Let X be a αrω–regular space. Let F be any closed set in X and a point $x \in X$ such that $x \notin F$. By [2] , F is αrω–closed and $x \notin F$. Since X is a αrω–regular space, there exists a pair of disjoint open sets G and H such that $F \subseteq G$ and $x \in H$. Hence X is a regular space.

Theorem 3.3. If X is a regular space and $T_{\alpha r\omega}$- space, then X is αrω– regular.

Proof: Let X be a regular space and $T_{\alpha r\omega}$- space. Let F be any αrω–closed set in X and a point $x \in X$ such that $x \notin F$. Since X is $T_{\alpha r\omega}$- space , F is closed and $x \notin F$. Since X is a regular space, there exists a pair of disjoint open sets G and H such that $F \subseteq G$ and $x \in H$. Hence X is a αrω–regular space

Theorem 3.4. Every αrω–regular space is α–regular.

Proof: Let X be a αrω–regular space. Let F be any α–closed set in X and a point $x \in X$ such that $x \notin F$. By [2] , F is αrω–closed and $x \notin F$. Since X is a αrω–regular space, there exists a pair of disjoint open sets G and H such that $F \subseteq G$ and $x \in H$. Hence X is a α–regular space.

We have the following characterization.

Theorem 3.5. The following statements are equivalent for a topological space X

i) X is a αrω– regular space.

ii) For each $x \in X$ and each αrω–open neighbourhood U of x there exists an open neighbourhood N of x such that $cl(N) \subseteq U$.

Proof: (i) => (ii): Suppose X is a αrω–regular space. Let U be any αrω–neighbourhood of x. Then there exists αrω–open set G such that $x \in G \subseteq U$. Now X – G is αrω–closed set and $x \notin X - G$. Since X is αrω–regular, there exist open sets M and N such that X – G ⊆ M , x ∈ N and M ∩N = ϕ and so N ⊆ X –M. Now cl(N) ⊆ cl(X – M) = X –M and X – G⊆ M. This implies X – M ⊆ G ⊆ U. Therefore cl(N) ⊆ U.

(ii) => (i): Let F be any αrω– closed set in X and x∉ F or x ∈ X –F and X –F is a αrω–open and so X –F is a αrω– neighbourhood of x. By hypothesis, there exists an open neighbourhood N of x such that x ∈ N and cl(N) ⊆ X –F. This implies F ⊆ X – cl(N) is an open set containing F and N ∩ {(X –cl(N)} = ϕ. Hence X is rw – regular space.

We have another characterization of αrω– regularity in the following.

Theorem 3.6. A topological space X is αrω–regular if and only if for each αrω–closed set F of X and each x ∈ X –F there exist open sets G and H of X such that x ϵ G, F ⊆ H and cl(G) ∩ cl(H) = ϕ.

Proof: Suppose X is αrω– regular space. Let F be a αrω–closed set in X with x ∉ F. Then there exists open sets M and H of X such that x ϵ M, F ⊆ H and M ∩H = ϕ. This implies M ∩ cl(H) = ϕ. As X is αrω–regular, there exist open sets U and V such that x ∈ U, cl(H) ⊆ V and U ∩ V = Φ, so cl(U) ∩ V = Φ. Let G = M ∩ U, then G and H are open sets of X such that x ϵ G, F ⊆ H and cl(H) ∩ cl(H) = ϕ.

Conversely, if for each αrω–closed set F of X and each x ϵ X– F there exists open sets G and H such that x ϵ G, F ⊆ H and cl(H) ∩ cl(H) = Φ. This implies x ϵ G, F ⊆ H and G ∩ H = ϕ. Hence X is αrω– regular.

Now we prove that αrω–regularity is a hereditary property.

Theorem 3.7. Every subspace of a αrω–regular space is αrω–regular.

Proof: Let X be a αrω–regular space. Let Y be a subspace of X. Let x ∈ Y and F be a αrω– closed set in Y such that x ∉ F. Then there is a closed set and so αrω–closed set A of X with F = Y ∩ A and x ∉ A. Therefore we have x ∈ X, A is αrω–closed in X such that x ∉ A. Since X is αrω–regular, there exist open sets G and H such that x ∈ G, A ⊆ H and G ∩ H = Φ. Note that Y ∩ G and Y ∩ H are open sets in Y . Also x ∈ G and x ∈ Y, which implies x ∈ Y ∩ G and A ⊆ H implies Y ∩ A ⊆ Y ∩ H, F ⊆ Y ∩ H. Also (Y ∩ G) ∩ (Y ∩ H) = Φ. Hence Y is αrω–regular space.

We have yet another characterization of αrω–regularity in the following.

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Theorem 3.8. The following statements about a topological space X are equivalent:

(i) X is αrω–regular

(ii) For each $x \in X$ and each αrω–open set U in X such that $x \in U$ there exists an open set V in X such that $x \in V \subseteq cl(V) \subseteq U$

(iii) For each point $x \in X$ and for each αrω–closed set A with $x \notin A$, there exists an open set V containing x such that $cl(V) \cap A = \Phi$.

Proof: (i)=> (ii): Follows from Theorem 3.5.

(ii) => (iii): Suppose (ii) holds. Let $x \in X$ and A be an αrω– closed set of X such that $x \notin A$. Then X –A is a αrω–open set with $x \in X - A$. By hypothesis, there exists an open set V such that $x \in V \subseteq cl(V) \subseteq X - A$. That is $x \in V$, $V \subseteq cl(A)$ and $cl(A) \subseteq X - A$. So $x \in V$ and $cl(V) \cap A = \Phi$.

(iii) => (ii): Let $x \in X$ and U be an αrω–open set in X such that $x \in U$. Then X –U is an αrω–closed set and $x \notin X - U$. Then by hypothesis, there exists an open set V containing x such that $cl(V) \cap (X - U) = \Phi$. Therefore $x \in V$, $cl(V) \subseteq U$ so $x \in V \subseteq cl(V) \subseteq U$.

The invariance of αrω– regularity is given in the following.

Theorem 3.9. Let $f : X \to Y$ be a bijective, αrω–irresolute and open map from a αrω–regular space X into a topological space Y , then Y is αrω–regular.

Proof: Let $y \in Y$ and F be a αrω–closed set in Y with $y \notin F$. Since f is αrω–irresolute, $f^{-1}(F)$ is αrω–closed set in X. Let $f(x) = y$ so that $x = f^{-1}(y)$ and $x \notin f^{-1}(F)$. Again X is αrω–regular space, there exist open sets U and V such that $x \in U$ and $f^{-1}(F) \subseteq G$, $U \cap V = \Phi$. Since f is open and bijective, we have $y \in f(U)$, $F \subseteq f(V)$ and $f(U) \cap f(V) = f(U \cap V) = f(\Phi) = \Phi$. Hence Y is αrω–regular space.

Theorem 3.10. Let $f : X \to Y$ be a bijective, αrω–closed map from a topological space X into a αrω–regular space Y . If X is $T_{\alpha r\omega}$-space, then X is αrω–regular.

Proof: Let $x \in X$ and F be an αrω–closed set in X with $x \notin F$. Since X is $T_{\alpha r\omega}$-space, F is closed in X. Then f(F) is αrω–closed set with $f(x) \notin f(F)$ in Y , since f is αrω–closed. As Y is αrω–regular, there exist disjoint open sets U and V such that $f(x) \in U$ and $f(F) \subseteq V$. Therefore $x \in f^{-1}(U)$ and $F \subseteq f^{-1}(V)$. Hence X is αrω–regular space.

4 αrω–Normal Spaces

In this section, we introduce the concept of αrω– normal spaces and study some of their characterizations.

Definition 4.1. A topological space X is said to be αrω–normal if for each pair of disjoint αrω– closed sets A and B in X, there exists a pair of disjoint open sets U and V in X such that $A \subseteq U$ and $B \subseteq V$.

We have the following interrelationship.

Theorem 4.2. Every αrω–normal space is normal.

Proof: Let X be a αrω–normal space. Let A and B be a pair of disjoint closed sets in X. From [2], A and B are αrω–closed sets in X. Since X is αrω–normal, there exists a pair of disjoint open sets G and H in X such that $A \subseteq G$ and $B \subseteq H$. Hence X is normal.

Remark 4.3. The converse need not be true in general as seen from the following example.

Example 4.4. Let Let X={a,b,c,d}, τ= {X, □, {a},{b},{a,b},{a,b,c}} Then the space X is normal but not αrω–normal, since the pair of disjoint αrω–closed sets namely, A = {c} and B = {d} for which there do not exists disjoint open sets G and H such that $A \subseteq G$ and $B \subseteq H$.

Theorem 4.4. If X is normal and $T_{\alpha r\omega}$-space, then X is αrω–normal.

Proof: Let X be a normal space. Let A and B be a pair of disjoint αrω–closed sets in X. since $T_{\alpha r\omega}$-space , A and B are closed sets in X. Since X normal, there exists a pair of disjoint open sets G and H in X such that $A \subseteq G$ and $B \subseteq H$. Hence X is αrω–normal.

Theorem 4.5. Every αrω–normal space is ω–normal.

Proof: Let X be a αrω–normal space. Let A and B be a pair of disjoint ω–closed sets in X. From [2], A and B are αrω–closed sets in X. Since X is αrω–normal, there exists a pair of disjoint open sets G and H in X such that $A \subseteq G$ and $B \subseteq H$. Hence X is ω–normal.

Hereditary property of αrω–normality is given in the following.

Theorem 4.6. A αrω–closed subspace of a αrω–normal space is αrω–normal.

Proof: Let X a be αrω–normal space. Let Y be a rw–closed subspace of X. Let A and B be pair of disjoint rw–closed sets in Y . Then A and B be pair of disjoint rw–closed sets in X. Since X is αrω–normal, there exist disjoint open sets G and H in X such that A ⊆ G and B ⊆ H. Since G and H are open in X, Y ∩ G and Y ∩ H are open in Y . Also we have A ⊆ G and B ⊆ H implies Y ∩ A ⊆ Y ∩ G , Y ∩ B ⊆ Y ∩ H . So A ⊆ Y ∩ G and B ⊆ Y ∩ H and (Y ∩ G) ∩ (Y ∩ H) = Y ∩ (G∩ H) = ϕ . Hence Y is αrω–normal.

We have the following characterization.

Theorem 4.7. The following statements for a topological space X are equivalent:

i) X is αrω–normal.
ii) For each αrω–closed set A and each αrω–open set U such that A ⊆ U, there exists an open set V such that A ⊆ V ⊆ cl(V) ⊆ U
iii) For any disjoint αrω–closed sets A, B, there exists an open set V such that A ⊆ V and cl(V) ∩ B = Φ
iv) For each pair A, B of disjoint αrω–closed sets there exist open sets U and V such that A ⊆ U, B ⊆ V and cl(U) ∩ cl(V) = Φ.

Proof: (i) => (ii): Let A be a αrω–closed set and U be a αrω–open set such that A ⊆ U. Then A and X –U are disjoint αrω–closed sets in X. Since X is αrω–normal , there exists a pair of disjoint open sets V and W in X such that A ⊆ V and X –U ⊆ W. Now X –W ⊆ X – (X –U), so X –W ⊆ U also V ∩W = Φ implies V ⊆ X –W, so cl (V) ⊆ cl(X –W) which implies cl(V) ⊆ X – W. Therefore cl(V) ⊆ X –W ⊆ U. So cl (V) ⊆ U. Hence A ⊆ V ⊆ cl(V) ⊆ U.

(ii)=>(iii): Let A and B be a pair of disjoint αrω– closed sets in X. Now A∩B = Φ, so A ⊆ X –B, where A is αrω–closed and X – B is αrω–open . Then by (ii) there exists an open set V such that A ⊆ V ⊆ cl(V)⊆ X –B. Now cl(V) ⊆ X –B implies cl(V) ∩ B = Φ. Thus A ⊆ V and cl(V) ∩ B = Φ

(iii) =>(iv): Let A and B be a pair of disjoint αrω–closed sets in X. Then from (iii) there exists an open set U such that A ⊆ U and cl(U)∩B = Φ. Since cl(V) is closed, so αrω–closed set. Therefore cl(V) and B are disjoint αrω–closed sets in X. By hypothesis, there exists an open set V , such that B ⊆ V and cl(U) ∩ cl(V) = Φ.

(iv) => (i): Let A and B be a pair of disjoint αrω–closed sets in X. Then from (iv) there exist an open sets U and V in X such that A ⊆ U, B ⊆ V and cl(U)∩cl(V) = Φ. So A ⊆ U , B ⊆ V and U ∩ V = Φ. Hence X αrω– normal.

Theorem 4.8. Let X be a topological space. Then X is αrω–normal if and only if for any pair A, B of disjoint αrω–closed sets there exist open sets U and V of X such that A ⊆U, B ⊆V and cl(U)∩cl(V)= Φ.
Proof: Follows from **Theorem 4.7.**

Theorem 4.9. Let X be a topological space. Then the following are equivalent:

(i) X is normal
(ii) For any disjoint closed sets A and B, there exist disjoint αrω–open sets U and V such that A ⊆ U , B ⊆ V .
(iii) For any closed set A and any open set V such that A ⊆ V , there exists an αrω–open set U of X such that A ⊆ U ⊆ αcl(U) ⊆ V .

Proof: (i) =>(ii): Suppose X is normal. Since every open set is αrω–open [2], (ii) follows.
(ii)=>(iii): Suppose (ii) holds. Let A be a closed set and V be an open set containing A. Then A and X –V are disjoint closed sets. By (ii), there exist disjoint αrω–open sets U and W such that A ⊆ U and X–V ⊆ W, since X –V is closed, so αrω–closed. From Theorem 2.3.14 [2], we have X –V ⊆ αint(W) and U ∩ αint(W) = Φ and so we have cl(U) ∩ αint(W) = Φ. Hence A ⊆ U ⊆ αcl(U) ⊆ X –αint(W) ⊆ V . Thus A ⊆ U ⊆ αcl(U) ⊆V.

(iii) =>(i): Let A and B be a pair of disjoint closed sets of X. Then A ⊆ X–B and X–B is open. There exists a αrω–open set G of X such that A ⊆ G ⊆ αcl(G) ⊆ X–B. Since A is closed, it is αrω–closed, we have A ⊆ int(G). Take U = int(cl(int(αint(G)))) and V = int(cl(int(X – αcl(G)))). Then U and V are disjoint open sets of X such that A ⊆ U and B ⊆ V . Hence X is normal.

Theorem 4.10. If f : X → Y is bijective , open ,αrω–irresolute from a αrω–normal space X onto Y then is αrω–normal.

Proof: Let A and B be disjoint αrω–closed sets in Y. Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint rw–closed sets in X as f is αrω–irresolute. Since X is αrω–normal , there exist disjoint open sets G and H in X such that $f^{-1}(A) \subseteq G$ and $f^{-1}(B) \subseteq H$. As f is bijective and open , f(G) and f(H) are disjoint open sets in Y such that $A \subseteq f(G)$ and $B \subseteq f(H)$. Hence Y is αrω–normal.

5 αrω–T_k Space (k=0,1,2)

Definition 5.1 A topological space X is called

i) a αrω–T_0 if for each pair of distinct points x, y of X, there exists a αrω–open sets G in X containing one of them and not the other.
ii) a αrω–T_1 if for each pair of distinct points x, y of X , there exists two αrω–open sets G_1, G_2 in X such that $x \in G_1$, $y \notin G_1$, and $y \in G_2$, $x \notin G_2$.
iii) a αrω–T_2 (αrω– Hausdorff) if for each pair of distinct points x, y of X there exists distinct αrω–open sets H_1 and H_2 such that H1 containing x but not y and H_2 containing y but not x.

Theorem 5.2

(i) Every T_0 space is αrω–T_0 space.
(ii) Every T_1 space is αrω–T_0 space.
(iii) Every T_1 space is αrω–T_1 space.
(iv) Every T_2 space is αrω–T_2 space.
(v) Every αrω–T_1 space is αrω–T_0 space.
(vi) Every αrω–T_2 space is αrω–T_1 space.

Proof: Straight forward.

The converse of the theorem need not be true as in the examples.

Example 5.3 Let X= {a, b, c, d} and τ ={X, Φ, {a},{b},{a, b},{a, b, c}}.
Then αrωC(X) = {Φ, X, {c}, {d},{a, d}, {b, d}, {c, d}, {a, c , d},{b, c, d},{a, b, d}}.
αrωO(X) ={ X, Φ, {a},{b},{c},{a, b},{a, c},{b, c},{a, b, c}{a, b, d}}.
Here (X, τ) is αrω–T_0 space but not T_0 space and not αrω–T_1 space.

Example 5.4 Let X= {a, b, c, d} and τ ={X, Φ, {a},{b},{a, b},{a, b, c}}.
Then αrωC(X) = {Φ, X, {c}, {d},{a, d}, {b, d}, {c, d}, {a, c , d},{b, c, d},{a, b, d}}.
αrωO(X) ={ X, Φ, {a},{b},{c},{a, b},{a, c},{b, c},{a, b, c}{a, b, d}}.
Here (X, τ) is αrω–T_1 space but not T_1 space and not αrω–T_2 space.

Theorem 5.5

(i) Every α–T_0 space is αrω–T_0 space.
(ii) Every α–T_1 space is αrω–T_0 space.
(iii) Every α–T_1 space is αrω–T_1 space.
(iv) Every α–T_2 space is αrω–T_2 space.

Proof: i) For each pair of distinct points x, y of X. Since α–T_0 space , there exists a α–open sets G in X containing one of them and not the other. But every α–open is αrω–open then there exists a α–open sets G in X containing one of them and not the other. Therefore αrω–T_0 space.

ii) Since α–T_1 space , but every α–T_1 space is α–T_0 space and also from Theorem5.5(i) . Therefore αrω–T_0 space.

iii) and (iv) similarly we can prove.

Theorem 5.6 Let X be a topological space and Y is an αrω–T_0 space. If f: X → Y is injective and αrω– irresolute then X is αrω– T_0 space.

Proof: Suppose x, y∈X such that x≠y. Since f is injective then f(x)≠f(y). Since Y is αrω–T_0 space then there exists a αrω–open sets U in Y such that f(x)∈U , f(y)∉U or there exists a αrω–open sets V in Y such that f(y)∈V , f(x)∉V with f(x)≠f(y). Since f is αrω–irresolute then $f^{-1}(U)$ is a αrω–open sets in X such that x∈$f^{-1}(U)$, y∉$f^{-1}(U)$ or $f^{-1}(V)$ is a αrω–open sets in X such that y∈$f^{-1}(V)$, x∉$f^{-1}(V)$. Hence X is αrω–T_0 space.

Theorem 5.7 Let X be a topological space and Y is an αrω–T_2 space. If f: X → Y is injective and αrω– irresolute then X is αrω– T_2 space.

Proof: Suppose x, y∈X such that x≠y. Since f is injective then f(x)≠f(y). Since Y is αrω–T_2 space then there are two αrω–open sets U and V in Y such that f(x)∈U , f(y)∈V and U∩V=Φ. Since f is αrω– irresolute then $f^{-1}(U)$, $f^{-1}(V)$ are two αrω– open sets in X, x∈$f^{-1}(U)$, y∈$f^{-1}(V)$,$f^{-1}(U)$∩$f^{-1}(V)$ =Φ. Hence X is αrω–T_2 space.

Theorem 5.8 Let X be a topological space and Y is an αrω–T_1 space. If f: X → Y is injective and αrω– irresolute then X is αrω–T_1 space.

Proof: Similarly to Theorem 5.7.

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Theorem 5.9 Let X be a topological space and Y is an T_2 space. If f: X → Y is injective and αrω– continuous then X is αrω–T_2 space.

Proof: Suppose x, y∈X such that x ≠ y. Since f is injective, then f(x)≠ f(y). Since Y is an T_2 space, then there are two open sets U and V in Y such that f(x)∈U, f(y)∈V and U∩V =Φ. Since f is αrω– continuous then $f^{-1}(U)$, $f^{-1}(V)$ are two αrω– open sets in X. Then x∈$f^{-1}(U)$, y∈$f^{-1}(V)$, $f^{-1}(U)$∩$f^{-1}(V)$ =Φ. Hence X is αrω–T_2 space.

Theorem 5.10 (X, τ) is αrω–T_0 space if and only if for each pair of distinct x, y of X, αrω–cl({x}) ≠ αrω–cl({y}) .

Proof: Let (X,τ) be a αrω–T0 space. Let x, y ∈ X such that x≠y, then there exists a αrω–open set V containing one of the points but not the other, say x∈V and y∉ V. Then V^c is a αrω–closed containing y but not x. But αrω–cl({y}) is the smallest αrω–closed set containing y. Therefore αrω–cl({y})⊂V^c and hence x∉ αrω– cl({y}). Thus αrω–cl({x}) ≠ αrω–cl({y}).

Conversely, suppose x, y ∈ X, x≠ y and αrω–cl({x}) ≠ αrω–cl({y}). Let z∈ X such that z ∈ αrω–cl({x}) but z ∉ αrω–cl({y}). If x ∈ αrω–cl({y}) then αrω–cl({x}) ⊂ αrω–cl({y}) and hence z∈αrω–cl({y}). This is a contradiction. Therefore x∉ αrω–cl({y}). That is x ∈ $(αrω–cl(y))^c$. Therefore $(αrω–cl(\{y\}))^c$ is a αrω– open set containing x but not y. Hence (X,τ) is αrω–T_0 space.

Theorem 5.11 A topological space X is αrω–T_1 space if and only if for every x∈ X singleton {x} is αrω– closed set in X.

Proof: Let X be αrω–T_1 space and let x∈X, to prove that {x} is αrω–closed set. We will prove X– {x} is αrω– open set in X. Let y ∈ X–{x}, implies x≠y ∈□ and since X is αrω–T_1 space then their exit two αrω– open sets G_1, G_2 such that x∉G_1, y∈ G_2 ⊆ X–{x}. Since y∈ G_2 ⊆ X–{x} then X–{x} is αrω– open set. Hence {x} is αrω–closed set. Conversely, Let x≠y ∈X then {x}, {y} are αrω– closed sets. That is X–{x} is αrω–open set. Clearly, x∉ X–{x} and y∈ X–{x}.Similarly X–{y} is αrω– open set, y∉X–{y} and x∈X–{y}. Hence X is αrω–T_1 space.

Theorem 5.12 For a topological space (X, τ), the following are equivalent

(i) (X, τ) is αrω–T_2 space.

(ii) If x∈X , then for each y≠ x , there is a αrω–open set U containing x such that y∉αrω–cl(U)

Proof: (i)⟹(ii) Let x∈X. If y∈X is such that y≠x there exists disjoint αrω–open sets U and V such that x∈U and y∈V. Then x∈U ⊂X–V which implies X–V is αrω– open and y∉X–V. Therefore y∉αrω–cl(U).

(ii) ⟹(i) Let x, y∈ X and x ≠y. By (ii) ,there exists a αrω– open U containing x such that y∉αrω–cl(U). Therefore y∈ X–(αrω–cl(U)). X–(αrω–cl(U)) is αrω–open and x∉X–(αrω–cl(U)). Also U∩X–(αrω– cl(U))= Φ. Hence (X,τ) is αrω–T_2 space.

Theorem 5.13 Let X be a topological space. If X is a αrω–regular and a T_1 space then X is an αrω–T_2 space.

Proof: Suppose x, y ∈X such that x≠y. Since X is T_1– space then there is an open set U such that x∈U, y∉U. Since X is αrω–regular space and U is an open set which contains x, then there is αrω–open set V such that x∈V⊂αrω–cl(V)⊆U. Since y∉U, hence y∉ αrω–cl(V). Therefore y∈X–(αrω–cl(V)). Hence there are αrω–open sets V and X–(αrω–cl(V)) such that (X– (αrω–cl(V)))∩V = Φ. Hence X is αrω–T_2 space.

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