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Accelerating and Decelerating Cosmological Models With Strange Quark Matter Attached to Cosmic String In General Relativity

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Abstract:-
In this paper we have studied the Riemannian space- time with quark matter attached to the cosmic string in general relativity. The Einstein's field equations have been solved. Also some physical and kinematical behaviors of the model thus obtained are discussed.

Keywords:- Strange quark matter, Cosmic strings ,Riemannian space time.

Subject Classification:- Subject classification 2015.

1. Introduction:

In this study, we have attached strange quark matter to the string cloud. It is plausible to attach strange quark matter to the string cloud. As one of such transition during the phase transition of the universe could be Quark Gluon plasma (QGP) harden gas(called quark-hadron phase transition) when cosmic temperature was $T \cong 200 MeV$. In the decade 1980's and 1990's, experiments at CERN's Super Proton Synchrotron (SPS) has credited to form the QGP; in 2000, the results led CERN to announce indirect evidence for a "new state of matter". Institutions such as Brookhaven National Laboratory's Relativistic Heavy Ion Collider (RHIC) are perusing this experiments**.** In Brookhaven National Laboratory [1,2,3], the quark-glucon plasma is created as a perfect liquid. There are many authors who contributed in study of quark matter and the relation between quark matter and domain walls and also string. Two ways of formation of strange quark matter have been proposed by Itoh [4], Bodmar [5] and Witten [6]**.** One of them is the quark – hadrons phase transition in the early universe and another one is conversion of neutron stars into strange ones at ultrahigh densities. In the theories of strong interaction, it is hypothysed that the breaking of physical vacuum takes place inside hadrons to form quark bag models. As a result, vacuum energy densities inside and outside a hadrons become essentially different and the vacuum pressure on the bag wall equilibrates the pressure of quarks thus making the system stabilized. If the hypothesis of the quark matter is true, some neutron stars could actually be stars built entirely of strange matters have been examined by Alcock et al. [7] and Haensel et al. [8]. Strange star properties have been studied by Cheng et al. [9] and Strange quark matter attached to the string cloud in spherical symmetric space-time admitting conformal motion have been investigated by Yavuz et al. [10]. The study of general relativistic treatment of strings have been proposed by Stachel [11] and Leterier [12]. The solutions of Einstein field equations for the string cosmological model with bulk viscous fluid obtained by D D Pawar [13]. In general relativity, The gravitational effects of cosmic strings have been investigated by Vilenkin [14], Gott [15]. Krori et al. [16], Banerjee and Bhui[17], Tikekar and patel [18], Bhattacharieee and Baruh [19] obtained relativistic string models of Bianchi space-time.

Cosmological models play a major role in the unfolding the secrets of the universe. The topological stable defects which occur during the phase transition are identified as strings by [20]. The other topological defects are monopoles and domain walls. The present day universe appears on astronomical considerations are of FRW type. A star which is smaller than neutron stars, the possibility of a quark star or a compact star, which is supported by degenerate pressure of quark matter, has been proposed before. Such a quark star has been investigated by many authors (Gerlach [21]; Ivanenko et al. [22]; Itoh). According to Rosenhauer et al. [23], such quark stars are in the branch of neutron stars, but, In Gerlach; Ivanenko et al.; and Itoh view, it is commonly assumed that, quark stars contain quark matter in the core region and are surrounded by harmonic matter. The formation of strange quark matter is the interesting consequences of the first order phase transition from quark phase to hadron phase in the early universe and it has been attracting much interst (Witten[6] ; Fahri and Jaffe [24]). It is plausible to attach strange quark matter to the string cosmology. As string is flexible by nature, different vibration models of the string represent the different particle types, since the different models are observed as different masses or spins.

The possibility of the existence of quark matter dates back to the early 1970s.Yilmaz et al. [25] obtained strange quark matter for the Roberts-Walker model in the context of the general theory of relativity. In the general theory of relativity, higher-dimensional Roberts-Walker cosmological models in the presence of quark-gluon plasma constructed by Yilmaz and Yavuz [26], also a Bianchi type-III cosmological model with strange quark matter attached to string cloud investigated by Adhav et al. [27]. Khadekar et al. [28] confined their work to the quark matter attached to the topological defects in general relativity. In general relativity, the geometry of quark and strange quark matter in higher dimensional investigated by Khadekar and Wanjari [29] and also axially symmetric space-time with strange quark matter attached to the string cloud studied by Katore and Shaikh [30]. In self-creation theory, Bianchi type-III cosmological model with strange quark matter attached to string cloud obtained by Mahanta et al. [31]. In Lyra geometry, Mahanta and Biswal [32] studied string cloud and domain walls with quark matter. In self-creation theory and general relativity, axially symmetric space-time with strange quark matter attached to the string cloud studied by Rao and Neelima [33]. In the Brans-Dicke theory of gravitation, axially symmetric space-time with strange quark matter attached to the string discussed by Rao and Sireesha [34]. In biometric theory, Sahoo and Mishra [35, 36] studied axially symmetric and plane symmetric cosmological solutions for quark matter coupled with string cloud and domain walls. Recently, Bianchi type –V cosmological model with strange quark matter attached to cosmic string investigated by Chirde and Rahate [37].

Typically quark matter is modeled with an equation of state which based on the phenomenological bag model of quark matter, in which quark confinement is described by an energy term proportional to the volume . In this model, quarks are through as degenerate Fermi gas, which exists only in a region of space endowed with a vacuum energy density B_c (called as the bag constant). In the framework of this model, the quark matter is composed of mass less u and d quarks, massive s quarks and electrons. In the simplified version of the bag model, it is assumed that quarks are mass less and non-interacting.

In this paper, we have studied the Riemannian space time with quark matter attached to the cosmic string in general relativity.

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2.The Metric And Field Equations:

We consider the Riemannian space time metric in the form

$$ds^2 = e^{2h}\left(dt^2 - dr^2 - r^2 d\theta^2 - s^2 dz^2\right) \tag{1}$$

Where r, θ, z are the usual cylindrical polar coordinates and $h \& s$ are functions of cosmic time t alone. It is well known that this line element is plane symmetric.

The energy momentum tensor for string cloud (Letelier 1983) is given by

$$T_{ij} = \rho u_i u_j - \rho_{s0} x_i x_j \tag{2}$$

Here ρ is the rest energy density for the cloud of strings with particles attached to them and ρ_{s0} is the string tension density. They are related by

$$\rho = \rho_p + \rho_{s0} \tag{3}$$

Where ρ_p is the particle energy density.

Therefore, we have quark pressure

$$p_q = \frac{\rho_q}{3} \tag{4}$$

Where ρ_q is the quark energy density. The total energy density is

$$\rho = \rho_q + B_c \tag{5}$$

And the total pressure is

$$p = p_q - B_c \tag{6}$$

We know that string is free to vibrate. The different vibration modes of the strings represent the different types of particles because these different modes are seen as different masses or spins. Therefore, here we will take quarks instead of particles in the string cloud. Hence we consider strange quark matter energy density instead of particle energy density in the string cloud. In this case from equation (3), we get

$$\rho = \rho_q + \rho_{s0} + B_c \tag{7}$$

From equation (2), and (7), (Yavuz et al. 2005) we have energy momentum tensor for strange quark matter attached to the string cloud as

$$T_{ij} = (\rho_q + \rho_{s0} + B_c) u_i u_j - \rho_{s0} x_i x_j \tag{8}$$

where u_i is the four velocity of the particles and x_i is the unit space like vector representing the direction of string.

We have u_i and x_i with satisfying conditions

$$u_i u^i = -x_i x^i = 1 \text{ and } u^i x_i = 0 \tag{9}$$

The Einstein's field equations (With gravitational units $8\pi G = C = 1$ read as

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j \tag{10}$$

Where R_i^j is the Ricci tensor and $R = g^{ij} R_{ij}$ is the Ricci scalar.

With the help of above equations, the field equations (10) for metric (1) can be written as follows:

$$2\ddot{h} + \frac{\ddot{s}}{s} + \dot{h}^2 + 2\dot{h}\frac{\dot{s}}{s} = 0 \tag{11}$$

$$\frac{1}{e^{2h}}\left[2\ddot{h} + \dot{h}^2\right] = \rho_{s0} \tag{12}$$

$$\frac{1}{e^{2h}}\left\{2\dot{h}\frac{\dot{s}}{s} + 3\dot{h}^2\right\} = -\rho \tag{13}$$

where the dot denotes ordinary differentiation with respect to t.

Hence we have three independent equations with four unknowns s , h , ρ and ρ_{s0}. Thus, these set of equations are nonlinear in nature. Therefore we need one assumption to get an exact solution of the field equations.

3. Solution Of The Field Equations:

To solve the above set of nonlinear field equations (11) – (13), We assume that relation between the metric potentials which is given as follows.

$$e^h = \beta s^n \tag{14}$$

Where β is constant and $n > 0$.

With the help of the equation (14), the solution of field equations (11) – (13), is given by

$$s = (at + b)^{\frac{2n+1}{(n+1)^2}} \tag{15}$$

$$e^h = \beta (at + b)^{\frac{n(2n+1)}{(n+1)^2}} \tag{16}$$

The Riemannian space time model with strange quark matter attached with cosmic string corresponding to equations (15) and (16), can be written as

$$ds^2 = \beta^2 (at+b)^{\frac{2n(2n+1)}{(n+1)^2}} \left\{ dt^2 - dr^2 - r^2 d\theta^2 - (at+b)^{\frac{2(2n+1)}{(n+1)^2}} dz^2 \right\} \tag{17}$$

Hence, the Riemannian space time model corresponding to the above solutions, through a proper choice of co-ordinates and constants of integration can be written as

$$ds^2 = \beta^2 T^{\frac{2n(2n+1)}{(n+1)^2}} \left\{ \frac{1}{a^2} dT^2 - dr^2 - r^2 d\theta^2 - T^{\frac{2(2n+1)}{(n+1)^2}} dz^2 \right\} \tag{18}$$

This model is free from singularity at initial epoch.
From equation (13), the string energy density is given by

$$\rho = -\frac{n(2n+1)^2 (3n+2) a^2}{\beta^2 (n+1)^4 T^{\frac{6n^2+6n+2}{(n+1)^2}}} \tag{19}$$

Using equation (12), we get the string tension density

$$\rho_{s0} = -\frac{n(2n+1)(3n+2) a^2}{\beta^2 (n+1)^4 T^{\frac{6n^2+6n+2}{(n+1)^2}}} \tag{20}$$

The string particle density is

$$\rho_p = -\frac{2n^2 (2n+1)(3n+2) a^2}{\beta^2 (n+1)^4 T^{\frac{6n^2+6n+2}{(n+1)^2}}} \tag{21}$$

Quark energy density is

$$\rho_q = -\frac{n(2n+1)^2 (3n+2) a^2}{\beta^2 (n+1)^4 T^{\frac{6n^2+6n+2}{(n+1)^2}}} - B_c \tag{22}$$

Quark pressure is

$$p_q = -\frac{n(2n+1)^2 (3n+2) a^2}{3\beta^2 (n+1)^4 T^{\frac{6n^2+6n+2}{(n+1)^2}}} - \frac{B_c}{3} \tag{23}$$

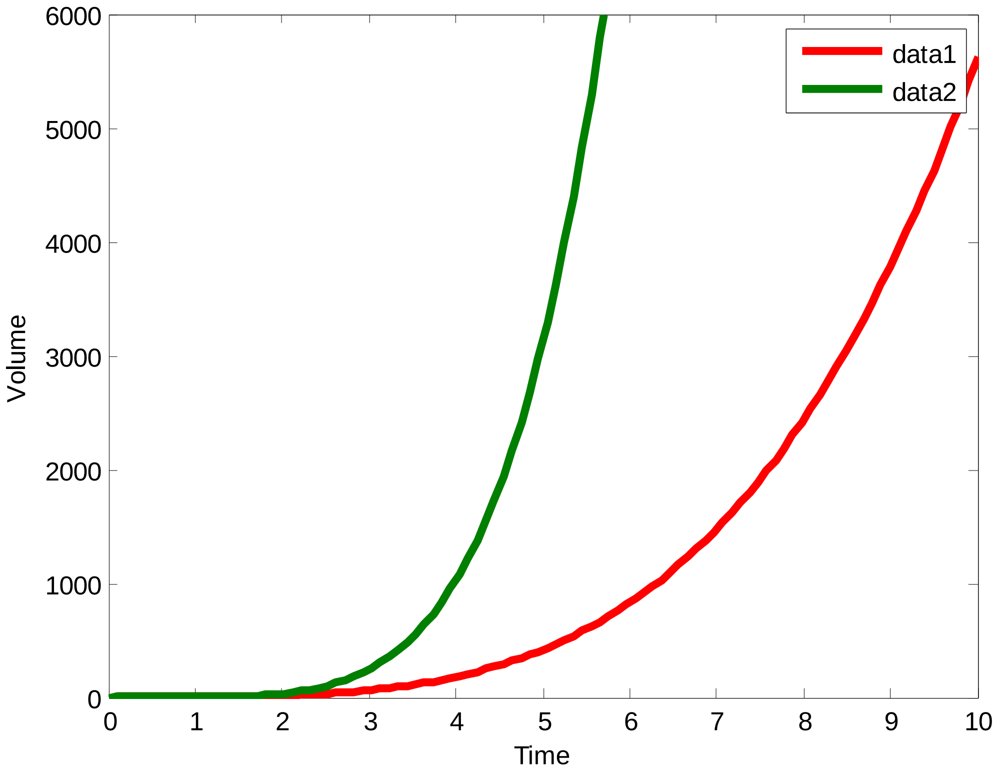
At initial epoch ($(T=0)$, quark pressure and density are infinite, further both decreases as T increases. Hence from equations (19) and (20), the energy density and tension density of the string possess initial singularities. However, as T increases these singularities vanishes.

4. Some Physical Properties:

In this model, the physical quantities spatial volume V^3 , the expansion scalar θ, shear scalar σ^2 are as given below:

Spatial volume is given by

$$V^3 = r\beta^4 T^{\frac{(2n+1)(4n+1)}{(n+1)^2}} \tag{24}$$



Data1: for n=.1

Data2: for n=2

It is observed here that, at an initial epoch $(T = 0)$, the proper volume will be zero, the spatial volume becomes infinitely large, for $T \to \infty$. Hence the model is expanding.

Scalar expansion θ is given by

$$\theta = u^i_{;i}$$

$$\theta = \frac{(2n+1)(4n+1)a}{(n+1)^2 T} \tag{25}$$

Hence expansion scalar tends to infinity as $T \to 0$, whereas the expansion scalar tends to zero, for $T \to \infty$.

The mean Hubble parameter H is given by

$$H = \frac{(2n+1)(4n+1)a}{3(n+1)^2 T} \tag{26}$$

The Shear scalar σ in model is given by

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}$$

$$\sigma^2 = \frac{7}{18}\frac{(2n+1)^2(4n+1)^2 a^2}{(n+1)^4 T^2} \tag{27}$$

Hence Hubble parameter and Shear scalar tends to infinity as $T \to 0$, whereas when $T \to \infty$, Hubble parameter and Shear scalar tends to zero.

Data1: Scalar expansion vs time

Data2: Hubble parameter vs time

Data1: Shear scalar vs time

Mean anisotropy parameter A_m is given by

$$A_m = \frac{2}{3}\frac{\sigma^2}{H^2}$$

$$A_m = \frac{7}{3} \neq 0 \tag{28}$$

Hence the model is anisotropy

The deceleration parameter is given by

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1$$

$$q = \frac{2-5n^2}{(2n+1)(4n+1)} \tag{29}$$

The deceleration parameter $q < 0$ for $n > \sqrt{\frac{2}{5}}$ and $q > 0$ for $0 < n < \sqrt{\frac{2}{5}}$.

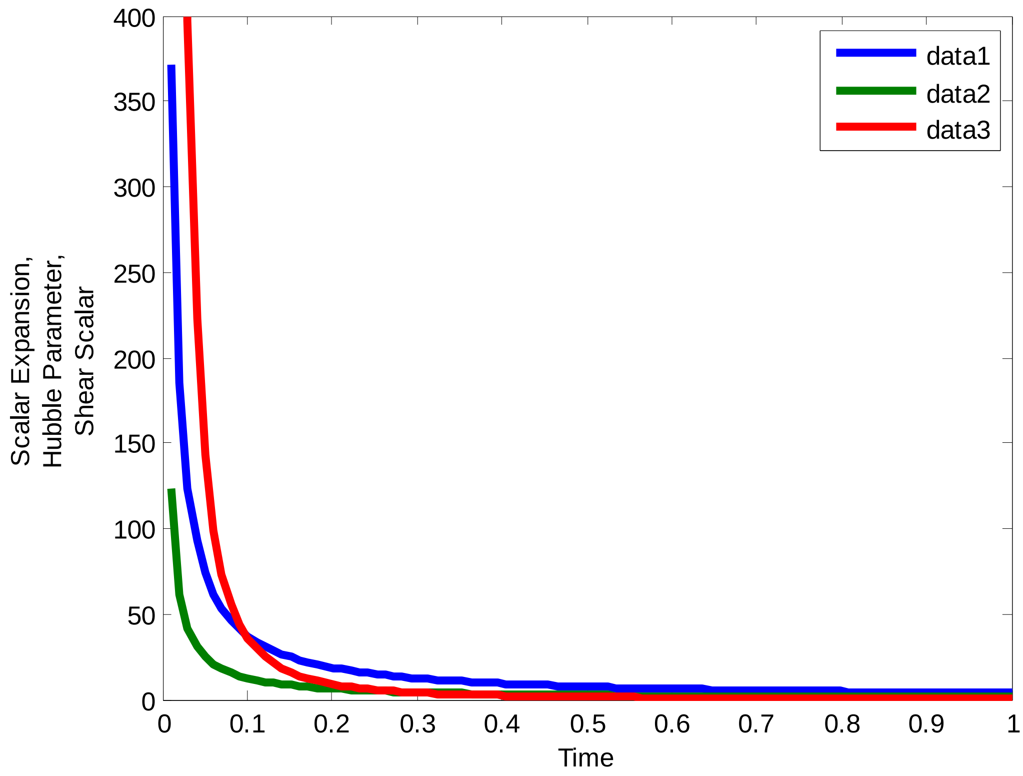
If $q < 0$, the model accelerates and when $q > 0$, the model decelerates in the standard way. Hence the model sometimes decelerates in the standard way and later accelerates which is in accordance with the present day scenario. However, in spite of the fact that the universe, in this case, decelerates in the standard way it will accelerate in finite time due to cosmic re callapse where the universe in turns inflates " decelerates and then accelerates" (Nojiri and Ordintsov 2003c).

The density parameter Ω is given by

$$\Omega = \frac{\rho}{3H^2} = \frac{-3n(3n+2)}{\beta^2 (4n+1)^2 T^{\frac{2n(2n+1)}{(n+1)^2}}} \qquad (30)$$

5.Conclusion:

In this paper we have presented the Riemannian space time with quark matter attached to the comic string in general relativity. The value of the mean anisotropic parameter is non-zero, therefore the nature of Universe is anisotropic. Also it is observed that the model is expanding, shearing, non-rotating, decelerating and accelerating. Also it is observed that for $n > \sqrt{\frac{2}{5}}$, the model accelerates , whereas the model decelerates for $0 < n < \sqrt{\frac{2}{5}}$.



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