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Fractal Analysis of time series and estimation of Hurst exponent in BSE

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Abstract

The estimation of Hurst Exponent for experimental data plays a very important role in the research that is in progress, which shows the characteristics of self-similarity. There are many techniques to predict the Hurst exponent using the time series. This paper presents comparative analysis of the statistical properties of the Hurst exponent estimators obtained by different methods using model stationary and non-stationary fractal time series. Random data share market data like BSE that possesses a high degree of variability and unpredictability can be analysed using R/S analysis and Hurst exponent H. Hurst exponent is revealing of the presence of noise or fluctuations in the data which in turn gives information on the volatility of the Close value and turbulence in the close value. The results of R/S analysis as applied to BSE indicate that there is persistence in the trends revealed by the BSE close value over the entire period studied as the value of H likes between 0.5 and 1.0. This also confirms long term memory effect that the trends tend to continue.

Keywords: Time series analysis, Hurst exponent, self- similarity, fractal analysis.

Subject Classification: Subject Classification 2015

1. Introduction

The progress of mathematical models, methods, and algorithms is required for the improvement and employment of modern electronics, radio electronics, control theory and image processing problems. Currently it has been generally accepted, many stochastic processes appear in nature and engineering in long-term dependence and fractal [1] structure. The most useful mathematical technique for mobility and the structure of such a process is a fractal analysis.In this paper the most commonly used method for estimating the Hurst exponents is examined. The various methods are: R / S -analysis, variance-time analysis, Detrended Fluctuation Analysis (DFA) and wavelet-based estimation [2].

Many years later, while investigating the fractal nature of financial markets noted

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mathematician Benoit Mandelbrot introduced to fractal geometry, in Hurst's honor, the term Generalized Hurst Exponent. The Hurst exponent is used as a measure of the long-term memory of a time series [3].

2 Estimating the Hurst exponent:

A variety of methodsoccur for estimating the Hurst exponent (H) and the process detailed here is both simple and highly data intensive. To estimate the Hurst exponent one must regress the rescaled range on the time span of observations. To do this, a time series of full length is divided into a number of shorter time series and the rescaled range is calculated for each of the smaller time series. A minimum length of eight is usually chosen for the length of the smallest time series. So, for example, if a time series has 128 observations it is divided into:

- two chunks of 64 observations each
- four chunks of 32 observations each
- eight chunks of 16 observations each
- 16 chunks of eight observations each

Steps for estimating the Hurst exponent after breaking the time series into chunks/masses:

For each chunk of observations, compute:

- the mean of the time series,
- a mean centered series by subtracting the mean from the series,
- the cumulative deviation of the series from the mean by summing up the mean centered values,
- the Range (R), which is the difference between the maximum value of the cumulative deviation and the minimum value of the cumulative deviation,
- the standard deviation (S) of the mean centered values, and
- the rescaled range by dividing the Range by the standard deviation.

Finally, average the rescaled range over all the chunks.

The rescaled range and chunk size follows a power law, and the Hurst exponent is given by the exponent of this power law. For example, the 80/20 rule (20 percent of the population holds 80 percent of wealth), the winner-take-all phenomenon, friend connections in a social network and forest fires all follow power laws.

Using the Hurst exponent we can classify time series into types and gain some insight into their dynamics. Here are some types of time series and the Hurst exponents associated with each of them [3].From the Hurst exponent H of a time series, the fractal dimension of the time series can be found. When Df=1.5, there is normal scaling , when Df is between 1.5 and 2, time series is anti-persistent and when Df is between 1 and 1.5 the time series is persistent. For Df =1.5, the time series is purely random. Long term correlations of indexes in developed and emerging markets have been studied by using Hurst analysis.

3Fractal dimension and Hurst exponent:

Fractal analysis is prepared by conducting rescaled range (R/S) analysis of time series. The Hurst exponent can classify a given time series in terms of whether it is a random, a persistent, or an anti-persistent process. Simulation study is run to study the distribution properties of the Hurst exponent using first-order autoregressive process. If time series data are randomly generated from a normal distribution then the estimated Hurst Exponents are also normally distributed [4]. The importance of the fractal dimension of a time series lies in the fact that it recognizes that a process can be somewhere between deterministic and random [5].

The Hurst Exponent is directly related to the fractal dimension, which measures the smoothness of a surface, or, in our case, the smoothness of a time series. The relationship between the fractal dimension D, and the Hurst Exponent H, is given by:

D=2-H……………………………….. [1]

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where, $0 \le H \le 1$. The closer the value of the Hurst Exponent to 0, the sharper will be the time series.

The Hurst Exponent is the measure of the smoothness of fractal time series based on the asymptotic behaviour of the rescaled range of the process.

The Hurst Exponent, H, can be estimated by:

$$H=\frac{\log\left(\frac{R}{S}\right)}{\log(T)} \quad \text{……………………….[2]}$$

where, T is the duration of the sample data and R / S the corresponding value of the rescaled range.

4 Rescaled Range Analysis:

Hurst (1965) established the rescaled range analysis, a statistical technique to analyze long records of natural phenomenon. Rescaled Range Analysis is the central tool of fractal data modeling. The two factors used in this range analysis are:

1) The difference between the maximum and the minimum cumulative values, and

2) The standard deviation from the observed values.

The R / S value scales as we increase the time increment, T, by a power law value which equals H, the Hurst Exponent. All fractals scale according to a power law. By rescaling data, we can compare diverse phenomena and time periods. Rescaled Range Analysis enables us to describe a time series that has no characteristic scale. Brownian motion is the primary model for a random walk process. Einstein (1908) found the distance a particle covers increases with respect to time according to the following relation:

$$R= T^{0.5} \quad \text{………………………….[3]}$$

Where, R is the distance covered by the particle in time T [6].

The theory of the rescaled range analysis was first given by Hurst. Mandelbrot and Wallis further refined the method. Feder (1988) gives an excellent review of the analysis of data using time series [7], history, theory and applications, and adds some more statistical experiments to establish the effectiveness of this approach. The parameter H, the Hurst exponent provides insight into the trends and patterns shown by the time series in respect of as to whether the time series is random or not. It is also related to the fractal dimension, the Rescaled range analysis (R/S) approach of estimating H is helpful in distinguishing a completely random time series from a correlated time series and. The value of H reveals persistence of trends in a given time series.

5 BSE SENSEX:

BSE Ltd (formerly Bombay Stock Exchange Ltd.), established in 1875, is Asia’s first and fastest Stock Exchange with the response time of 200 microseconds and one of India’s leading exchange groups and has played a prominent role in developing the Indian capital market. BSE is a corporate and demutualized entity, which has a broader shareholder base, in which two leading global exchanges, Deutsche Bourse and Singapore Exchange are strategic partners. BSE offers an efficient and transparent market for equity, debt instruments, equity derivatives, currency derivatives, interest rate derivatives, mutual funds and stock lending and borrowing. There is also a dedicated platform for trading in Small and Medium Enterprises (SMEs) Equities in BSE. BSE offers many other services of market participants including risk management, clearing, settlement, market data services and education.It has a global reach with customers and a nation-wide presence.BSE systems and processes are designed to protect the integrity of market, to stimulate Indian capital market growth and to stimulate innovation and competition in all market segments [8].

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Stock market index used by Bombay Stock Exchange. The S&P BSE SENSEX (S&P Bombay Stock Exchange Sensitive index) also called the BSE 30 or simply the SENSEX. This is a free float market – weighted stock market index of 30 well established and financially sound companies listed on Bombay Stock Exchange. The 30 component companies which are some of the largest and most actively traded stocks, are representative of various industrial sectors of the Indian economy.

The time series shown next (Figure 1) was taken for the purpose of estimating the Hurst Exponent.

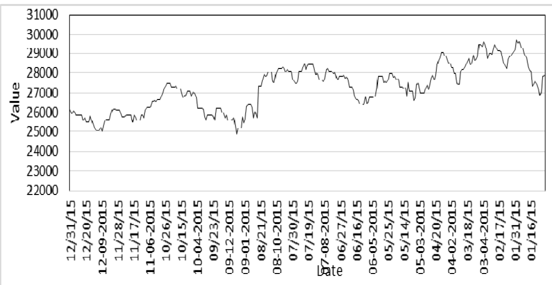

Figure1 : Time series of 2015

SNo	τ	τ /2	MEAN R/S	LOG (τ /2)	LOG(R/S)
1	4	2	0.901455	0.30103	-4.51E-02
2	8	4	1.797278	0.60206	0.254615
3	16	8	3.419816	0.90309	0.534003
4	32	16	6.051977	1.20412	0.781897
5	64	32	12.04029	1.50515	1.080637
6	128	64	20.80745	1.80618	1.318219
7	256	128	34.06506	2.10721	1.532309

Table 1: BSE 2015

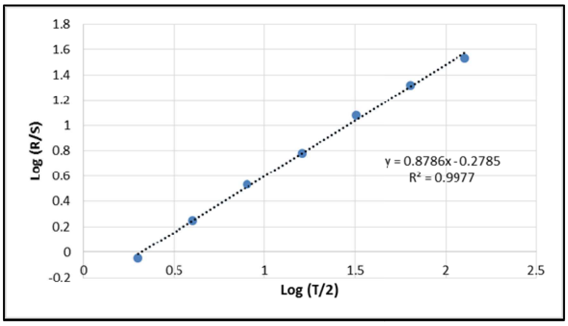

Figure2:Plot of log(τ /2) vs log (R/S)

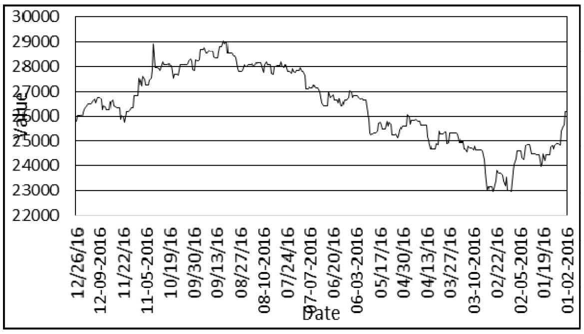

Figure3 : Time series of 2016

The above figure 3 shows the time series of Bombay sensex index for the year 2016

Sr.No	τ	τ /2	MEAN R/S	LOG (τ /2)	LOG(R/S)
1	4	2	0.901455	0.30103	-4.51E-02
2	8	4	1.797586	0.60206	0.25469
3	16	8	3.464548	0.90309	0.539647
4	32	16	6.274471	1.20412	0.797577
5	64	32	12.52975	1.50515	1.097942
6	128	64	26.96609	1.80618	1.430818
7	256	128	55.83295	2.10721	1.746891

Table 2: Data points for BSE 2016

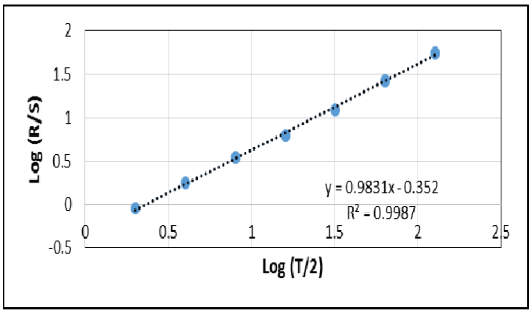

Figure 4 :Plot of log(T/2)vs log (R/S)

6 Interpretations from the Hurst Exponent:

The value of Hurst Exponent varies between 0 and 1. H = 0.5 indicates a random walk or an independent process. If $0 \leq H \leq 0.5$ then we have antipersistence. Such a process covers less distance than a random walk. This means that a decreasing process or a time series, then, it is more probable that the process or the time series will show an increasing trend. An anti-persistent time series will exhibit higher noise and more volatility. If $0.5 < H \leq 1$ then we have persistence. A persistent process or a time series will cover more distance than a random walk. This means that a decreasing process or a time series, then, it is more probable that the process will continue to decrease, and if we have an increasing time series, then, it is more probable that the time series will continue to show an increasing trend. A persistent time series has long memory effects. In theory, the trend at a particular point in time affects the remainder of the time series. A persistent time series will show higher noise and more instability.

The relationship between fractal dimensions D f and Hurst exponent H can be expressed as

D f = 2 – H

From the Hurst exponent H of a time series, the fractal dimension can be found from this equation. When D f = 1.5, there is normal scaling. When D f is between 1.5 and 2, time series is anti-persistent and when D f is between 1 and 1.5 the time series is persistent. For D f =1, time series is a smooth curve and purely deterministic in nature and for D f = 1.5 time series is purely random.

7 R/S analysis of BSE sensex time series:

The BSE sensexdata spread over about two years is split into two parts with 350 data points in each set (Set – I to II) For the purpose of implementation of R/S analysis. Set – I for 2015 and set – II is for 2016. This covers a period from 13/01/2015 to 26/12/2016; record of BSE sensex indices[9] is used. Figure 1 and 3 is the complete plot showing the changing BSE indices over two years, the period under study. There was appreciable rising trend in the initial part. To explore the reality, instead of Date versus close value a plot of log of date versus value is shown in Figure 2, it is seen from this plot that rising trend persist over the entire period and is superimposed with fluctuations. The Set – I representing the relatively faster rising trend compared to the Set-II. The R/S analysis was implemented individually on each set taking one set at a time [10-16].

The results of R/S analysis implemented individually on each sets (Set – I and II) taking one set at a time is presented in Table 1 First column is the log(T/2) value and the remaining four columns show the corresponding values of log(R/S) for each set. The log log plot of R/S against T/2 is shown in Figure 4, it is interesting to note that all the plots for the two sets lie close to each other showing identical trend and almost superimpose. The slope of the straight line best fitting the data point lie between 0.5 and 1.0, in fact more than 0.9, signifying persistent trend entailing that the rising value continue to rise and the falling value continue to fall that is in agreement with what is seen from the plots, of course there is always a variability and there are turning points where the close values display the trend reversal and that is one point or two and rest of the points follow certain trend [17].

Table 6: Mean R/S and T/2 for BSE 2015 - 2016

	MEAN R/S	
LOG (τ /2)	SET-I 2015	SET-II 2016
0.30103	-0.0450558	-0.0450558
0.60206	0.25461518	0.25468961
0.90309	0.53400271	0.5396466
1.20412	0.78189725	0.7975771
1.50515	1.08063691	1.0979423
1.80618	1.31821877	1.430818
2.10721	1.53230911	1.74689055

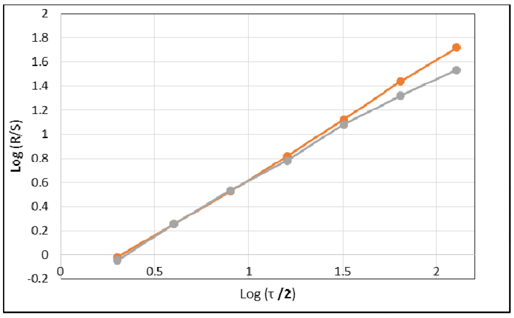

Figure 5: Gives Rescale Range Analysis of 2 Sets of BSE sensex Analysis of 1335 Data Points

Table 7: Fractal dimension calculated from the Hurst exponent

Sr.No.	Set	Hurst Exponent H	Fractal Dimension Df	R2
1	I	0.8786	1.1214	0.9977
2	II	0.9831	1.0169	0.9987

The value of the Hurst exponent estimated from the slope of the graph and the fractal dimension calculated from the Hurst exponent are tabulated in Table 7 the values of R2 are shown in the last column which indicated the least square fit line best describes the data as the points lie well along this straight line and the value of R2 is close to unity. The value of Hurst exponent H ranges approximately from 0.88 to 0.98 and fractal dimension between 1 and 1.5 confirming the persistence in trends, meaning that a rising trend is precursor of further rise. The fractal dimension D f for Set – I and II are on the lower side indicating relatively less fluctuations and complexity of trend as compared to the remaining two sets.

8 Result and Conclusion:

Random data share market data like BSE that possesses a high degree of variability and unpredictability can be analysed using R/S analysis and Hurst exponent H. Hurst exponent is revealing of the presence of noise or fluctuations in the data which in turn gives information on the volatility of the Close value and turbulence in the close value. The results of R/S analysis as applied to BSE indicate that there is persistence in the trends revealed by the BSE close value over the entire period studied as the value of H likes between 0.5 and 1.0. This also confirms long term memory effect that the trends tend to continue. As the value of H range from about 0.88 to 0.98, it indicates that H being on the higher side tending to unity, imply that the time series lean towards deterministic nature and show limited roughness or variability. A closer examination of the four sets of data and their respective H and D values shows that Set – II has the highest value of H and Set – I has the lowest.Both sets clearly confirm persistence of the trend and existence of long term memory effect with limited degree of local fluctuations.

9References

1. Zakde K. R1, Munde S.V2, Yusuf H.Shaikh, Journal of Chemical, Biological and Physical Sciences,An International Peer Review E-3 Journal of Sciences,May 2014 – July 2014, Vol. 4, No. 3; 3587-3592. E- ISSN: 2249 –1929
2. LudmilaKirichenko, Tamara Radivilova, ZhannaDeineko, International Journal "Information Technologies & Knowledge" Vol.5 / 2011,pp 371
3. Mu Sigma , White paper , Predictability of Time Series, July 2012
4. MalharKale,FerryButar,Journal of Mathematical Sciences & Mathematics Education, Vol. 5 No. 1

5. Peters, E. E. (1996). Chaos and Order in the Capital Markets (2nd ed.), John Wiley & Sons, Inc. Peters, E. E. (1994). Fractal Market Analysis: Applying Chaos Theory to Investment and Economics, John Wiley &Sons, Inc.
6. Peters E. E. (1994). Fractal Market Analysis: Applying Chaos Theory to Investment and Economics, John Wiley &Sons, Inc.
7. Kantelhardt, B. E Koscielny., H. H Rego A, S Havlin, Bunde A, Physica A 295, 441(2001).19. T.C. Mills, 'Statistical analysis of daily gold price data' PhysicaA 338 (3–4), 559–566(2004).
8. http://www.bseindia.com
9. http://www.bseindia.com/indices/IndexArchiveData
10. Razdan 'Scaling in the Bombay stock exchange index' Pramana58, 3,537(2002).
11. H.E Hurst., 'Long-term storage capacity of reservoirs' Trans. Am. Soc. Civil Eng'116, 770 (1951).
12. Indian Stock Market Research: 5 years Fact sheets and Analysis http://www.equitymaster.com/research-it/
13. X. Zhou, N. Persaud, H. Wang. 'Scale invariance of daily runoff time series in agricultural watersheds 'Hydrology and Earth System Sciences 10, 79(2006).
14. A.R Rao., D. Bhattacharya, 'Comparison of Hurst Exponent Estimates in Hydrometeorological Time Series' J. of Hydrologic Eng, 4, 3, 225,(1999).
15. Granger, C. W. J, 'Some properies of Time Series Data and Their Use in Econometric Specification'. Journal of Econometrics; 16: 121-130(1981).
16. Dr. S. Kaliyamoorthy, 'Relationship of Gold Market and Stock Market: An Analysis' International Journal of Business and Management Tomorrow Vol. 2 No. 6 (2012).
17. Capie, F., Terence, C.M., Wood, G., 'Gold as a hedge against the dollar' 'International Financial Markets,Institution and Money 15, 343-352(2005).